Assessing Students’ Number Sense: What to Be Considered?

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Abstract: Since the notion of number sense is remain unclear among researchers, the recent study intends to provide a literature-based understanding of the key concepts of number sense and how to assess the notion. Although researchers define the term ‘number sense’ in many different ways, they somehow agree that the term generally depicts someone’s proficiency in dealing with numbers and number operations in computational situation or problems. People who have an adequate sense of numbers possess: (1) an appropriate understanding of numbers and how they are related one to another; (2) an understanding the meaning of operations and how they are related and impact numbers or other operations; (3) a capability performing computation by utilizing their understanding of the numbers and the operations fluently and flexibly in dealing with number-related problems including making reasonable estimation; and (4) a faculty in making appropriate judgment of calculation including identifying potential error of a computation and making estimation. Therefore, assessing students’ number sense means assessing their faculties on the four aspects.

1 INTRODUCTION

The term number sense is used to depict a complex idea referring to the extent of people aptitude in working with numbers and their manipulation (operations). Due to its complexity, there is no any single definition yet that are used among scientists to represent the term ultimately. It is considered a holistic construct that is not easy to define in a single definition (Jordan & Dyson, 2016; Yang & Wu, 2010).

However, scientists strongly agree that the faculty in dealing with numbers will determine people aptitude in the higher aspects of mathematics and also determine their future career (Jordan & Dyson, 2016; Wu, 1999; Jordan et al., 2013; Vukovic et al., 2014; Ontario, 2013; Cai & Knuth, 2011; Bill, et.al. 2010; Hope & Sherrill, 1987; Bobis, 1991; Case & Sowder, 1990; Cobb, et.al., 1991; Jordan et al., 2009). It is claimed that students’ lack competency in dealing with numbers affects their achievements in the other aspects of mathematics, such as working with fractions and algebra (Wu, 1999; Jordan et al., 2013;Vukovic et al., 2014), dealing with algebraic problems (Ontario, 2013; Cai & Knuth, 2011; Bill, et.al. 2010), doing mental calculation and estimation (Hope & Sherrill, 1987; Bobis, 1991; Case & Sowder, 1990), and performing problem solving (Cobb, et.al., 1991; Jordan et al., 2009). In another aspect, students’ level of number sense can be used as the basis to predict students’ achievement in higher aspects of mathematics (Robinson, Menchetti, & Torgesen, 2002).

Considering the potential impacts, assessing students’ number sense is crucial to prevent them from the failure in mathematics. In this case, Jordan & Dyson (2016) stress that screening students’ aptitude towards numbers can be used as the basis to determine students who need more assistance to prevent them from failure in learning mathematics.

However, what and how to assess students’ number sense is still debatable among scientists due to its broad and complex ideas (Jordan & Dyson, 2016). Therefore, the current article intends to discuss the key concepts of number sense and how to assess the notion.

2 METHOD

Since the term number sense refers to complex ideas relating to human capability in dealing with numbers

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and number manipulations, there is no single definite definition among scientists to describe the term. For example, among psychologists, number sense is defined as the ability to recognize, understand, estimate and work with numbers regarding non-symbolic ideas of numbers (see Dehaene, 2011). Meanwhile, educators tend to regard number sense as the faculty in dealing with not only non-symbolic aspects of numbers but also the symbolic one in number-related problem solving (see NCTM, 2000 or Fosnot & Dolk, 2001). The slightly different views among the two domains of scientists may contribute to confusion among researchers and educational practitioners, especially those beginners, in conceiving and assessing students’ number sense.

Regarding the proposition above, the current article intends to build a robust literature-based explanation about the definition and considerations in assessing students’ number sense. There are three main intentions of the study, such as (1) constructing literature-based understanding relating to the views among scientists about number sense; (2) clarifying elements in assessing students’ number sense; and (3) identifying considerations in assessing students’ number sense.

There are seven sources of qualified literature which are studied intensively to identify their definitions, key concepts, and characteristics of number sense to uncover potential similarities and differences. The similarities and differences then are studied critically in order to identify their common tendencies. The literatures are from various type of publications (such as journal articles, books, book chapters, and a proceeding) which is publisher by qualified publishers (including Elsevier Academic Press, Oxford University Press, other institutional publishers) from various countries (like The United State of America, The United Kingdom, Sweden, and Indonesia) and recent years of publications (range from 2000 to 2018).

Before concluding, the initial findings are presented and discussed among experts and educators to be criticized to gain a robust conception about the object of the study. The conclusion is discussed in the recent article.

3 FINDING AND DISCUSSION

In this part, various definitions of number sense, its key concepts and its indicators of assessments are discussed which based on several related and qualified literature from several different resources and recent years of publications.

3.1 Definitions of Number Sense

No doubt among the scientists that students’ sense of numbers will determine their aptitude in the higher aspects of mathematics and also determine their future career (Jordan & Dyson, 2016; Wu, 1999; Jordan et al., 2013; Vukovic et al., 2014; Ontario, 2013; Cai & Knuth, 2011; Bill, et.al. 2010; Hope & Sherrill, 1987; Bobis, 1991; Case & Sowder, 1990; Cobb, et.al., 1991; Jordan et al., 2009). Regarding the significant impact of number sense, it is necessary to have a clear and appropriate understanding of the concepts underpinning the terms.

The term ‘number sense’ is a complex concept that does not easy to define precisely. In general, the extent of students’ flexibility in dealing with quantitative ideas about numbers, their relationships and as well as their manipulations indicates students’ sense of numbers. There are various definitions used to express such a kind of sense. Jordan, Fuchs & Dyson (2014), for example, merely define number sense as children knowledge of numbers, number relations, and number operations. Meanwhile, Andrews & Sayers (2014) relate number sense as the ability to operate flexibly with number and quantity.

Some other researchers assert number sense in the broader context that seeing number sense is not only a form of knowledge (which to possess or not) but also a type of problem-solving expertise that develops over time as the result of experiences. Fosnot & Dolk (2001), for example, consider number sense as proficiency in grasping the notions of numbers and their manipulations in emerging and selecting efficient counting strategies in problem-solving contexts. Fosnot & Dolk (2001) stress that students’ senses of numbers are developed over time through arbitrary or nonlinear learning trajectories (as opposed to linear learning trajectories that are traditionally practiced in mathematics instructions) as the result of intended or hypothesized learning experience. Here, students’ development of senses of number forms a landscape of learning containing some big ideas (such as cardinality and magnitude of numbers), strategies of reasoning (such as one-to-one tagging, skip counting, and counting on), and models of thinking (such as tallies and number lines). The landscape of learning is not developed instantly but it is developed over time through hypothesized learning trajectories (see Simon’s idea of hypothetical learning trajectories (Simon, 1995)).

Moreover, Fosnot & Dolk (2001) assert that calculating with number sense means the ability to select appropriate and efficient counting strategies by considering the characteristic of the numbers and the problems. To be able to perform in this level, students are required to have a well-established understanding of numbers, operations, and the relationship between and among them. Then, those understanding are utilized in emerging efficient counting strategies flexibly in solving number-related problems. To easily understand the idea, Fosnot & Dolk (2001) confront between counting by numbers sense and non-number sense as they said “Using algorithms, the same series of steps with all problems, is antithetical to calculating with number sense. Calculating with number sense means that one must look at the numbers first and then decide on a strategy that is fitting—and efficient.” (Page 124).It means that one a student has appropriated or adequate number sense, he/she does not rely on a specific counting procedure, but he/she has flexibility in emerging and selecting a counting strategy that seems more efficient to tackle computation problems.

Some scientists elucidate number sense in a more operational definition. NCTM (2000), for example, describe students who proficient in numbers as they can decompose numbers naturally, use particular numbers as referents, use the relationships among arithmetic operations to solve problems, understand the base-ten number system, estimate, make sense of numbers, and recognize the relative and absolute magnitude of numbers. NCTM (2000) proposes that teaching for numbers and operations should focus on building students’ number sense consisting of three competencies. The first is the understanding of meaning the numbers and the relationships among them. The second is the comprehending with number operations and how they are related one to another. The third is handy in computation and estimating by employing their understanding of numbers and operations. Therefore, broadly speaking, NCTM (2000) defines students’ number sense as students’ ability in utilizing their understanding of numbers and operations in dealing with computational problems.

Other researchers relate students’ number sense as the ability to think logically, critically and creatively in dealing with computational problems. Putrawangsa & Hasanah (2018), for example, consider number sense as the ability to utilize logical, critical and creative reasoning and understanding about numbers and operations in dealing with number-related problems flexibly, effectively, efficiently and practically. Once students possess an adequate sense of number, they can flexibly formulate by themselves several counting strategies and logically and critically select one of those that fit and efficient with the problems. In line with Fosnot & Dolk (2001), Putrawangsa & Hasanah (2018) stress that students who possess a sense of number do not rely on a specific counting procedure or an algorithm, but they can emerge various counting strategies or counting approaches to handling a particular counting problem.

Dissimilar from other researchers, Ali (2014) incorporates the notion of number sense with the ability to produce mathematical judgment about numbers and their operations. Ali (2014) stresses that students with good number sense can use numbers in flexible ways to make a mathematics-based judgment and to develop useful strategies for handling problems relating to numbers and operations. Here, students who have a sense of numbers can compose their procedures in doing the calculation and recognize significant numerical errors of a particular calculation. They own a good sense of numerical magnitude which helps them in represent the same number in multiple ways. Ali (2014), moreover, asserts that students with good number sense have the ability to use numbers in flexible ways to make mathematical judgment and to develop useful strategies for handling numbers and operations

Considering the various depictions of number sense among scientists describe above, all researchers relate number sense to the idea about numbers and their manipulations (operations). However, there are some varieties among them in expressing the focus and the scope of the idea. Some of them relate it to the context of problem-solving, making mathematical judgment, or dealing with computation. Hence, it can be deduced that the notion of number sense is used to describes the extent of students’ proficiency in dealing with quantitative ideas of numbers, their relationships, and their manipulation (operations) in context of solving number-related problems including making estimation and number-related judgment. Students who have a strong sense of numbers can be characterized as the following evidences: (1) own understanding about numbers, their manipulations (operations), and the relationships among them;(2) unbounded to a certain counting procedure or algorithm; (3) being able to generate various counting strategies by their own flexibly in dealing with number-related problems; (4) being able to make number-based estimation and judgment. The skill develops over the time as the result of the development of students’ understanding of numbers, their manipulations (operations), and the relationships among them acquired from life experiences, both educational and non-educational experiences.

3.2 Various Perspectives toward the Domain of Number Sense

It is identified that there are two main perspectives regarding numbers which shape the scope of the discussion about number sense, such as non-symbolic and symbolic aspects of numbers (Sokolowski & Ansari, 2016). The non-symbolic aspects refer to human aptitude about quantitative ideas of numbers without considering written or verbal representations of numbers (symbolism). For instance, children who are still illiterate can recognize one box which contains more marbles. This proficiency is human inherent capacity, and it is developed naturally. Meanwhile, symbolic aspects of numbers refer to human faculty in working with quantitative ideas of numbers that implicate numerical representations either in the form of written or verbal expression. For example, children know that five is used to represent five things or understand that the expression 5 – 2 means five is subtracted by two. People who grasp advanced concepts about symbolic ideas of numbers can easily recognize the meaning of complicated numerical symbolization. Brain representations that underlie non-symbolic numbers are considered innate in the human brain and consequently are not necessary to be constructed through learning interventions (Cantlon, 2012). In contrast, brain representations that underlie symbolic numbers are not innate, and therefore they must be developed through learning interventions (Ansari, 2008).

Regarding this symbolization, number sense has defined the difference between psychologists and educators slightly. Cognitive psychologists tend to consider number sense as the ability to recognize, understand, estimate and work with non-symbolic ideas of numbers (Dehaene, 2011). Meanwhile, educators tend to regard number sense as the capacity to deal with both non-symbolic and symbolic ideas of numbers (NCTM, 2000; Fosnot & Dolk, 2001). NCTM (2000), for example, stresses that understanding numbers and their symbolizations are the basis for developing students' number sense. The similar findings also coined by Berch (2005) that psychologists and mathematics educators work to different definitions of number sense. However, both groups of researchers agree that developing students' number sense in early grades is crucial since it plays important role as foundational concepts for students to master further advance concepts of mathematics (Jordan et al. 2013) and potentially determine students' opportunity in the future career (National Mathematics Advisory Panel, 2008).

Another perspective relating to number sense is the one proposed by Jordan & Dyson (2016). They use the term verbal and non-verbal to refer two aspects of number sense. The term verbal knowledge of numbers refers to students understanding of symbolic aspects of numbers. Meanwhile, for the non-symbolic aspects, they coin the term non-verbal knowledge of numbers. They argue that both verbal and non-verbal knowledge of numbers is prominent to support students' sense acquisition of numbers.

If the previous perspectives of numbers sense differentiate number sense regarding number representations. Andrews and Sayers (2014) propose a distinct perspective in looking at number sense which is based on its utility. Andrews and Sayers (2014) propose the term foundational and applied number sense. The foundational number sense is defined as students' early number-related understanding as the result of early formal instructions. Meanwhile, the term applied number sense indicates students' proficiency in utilizing their foundational number sense to function effectively in society. According to Andrews and Sayers (2014), the foundational sense of numbers at least comprise seven aspects, such as (1) involving number cognition, its vocabulary and its meaning; (2) incorporating systemic counting, including the notion of cordiality and cardinality; (3) Recognizing the relationships between numbers and quantity; (4) Recognizing the magnitudes of numbers and the comparisons between those magnitudes; (5) Recognizing and making estimation of numbers; (6) Being able to perform simple arithmetic operations, such as addition and subtraction; and (7) Recognizing of number patterns. Other than those seven aspects may be considered as applied number sense. Students who own an excellent applied number sense will consider the relationship among operated numbers and the operations in inventing and selecting counting strategies, consider the context of the problem in inventing the strategies, recognize the unreasonable answer, and recognize the need for making the estimation. However, the differences between foundational and applied number sense remain vague.

Researchers' and educators' view toward number sense definition and perspective will determine their approach of assessing students' number sense.

3.3 Indicators of Number Sense Assessment

Many researches identify that students' number sense will determine their mastery in the higher aspects of mathematics and also determine their future career

(Jordan & Dyson, 2016; Wu, 1999; Jordan et al., 2013; Vukovic et al., 2014; Ontario, 2013; Cai & Knuth, 2011; Bill, et.al. 2010; Hope & Sherrill, 1987; Bobis, 1991; Case & Sowder, 1990; Cobb, et.al., 1991; Jordan et al., 2009). Therefore, assessing students' number sense is crucial to prevent them from the failures. However, what are the considerations once assessing students' number sense? This question will be addressed in this part.

According to NCTM (2000), the central core of teaching numbers and operations is teaching for number sense. Once students acquire an adequate sense of numbers, they can decompose numbers, use particular numbers as referents, use the relationships among arithmetic operations to solve problems, understand the base-ten number system, make an estimation, and recognize the relative and absolute magnitude of numbers.

NCTM (2000) formulates several main competencies about numbers and operations that comprise students' number sense. Those are understanding the meaning of numbers, ways of representing numbers, recognizing the relationships among numbers, number systems, understanding the meanings of operations, how the operations are relate done to another; being able to compute fluently and making reasonable estimation in computation. Those competencies can be categorized into three aspects. The first aspect is dealing with understanding numbers including the relations among the numbers. The second is the understanding of operations and how they are related one to another. The last aspect is relating to skillfulness in performing computations fluently and flexibly by utilizing their understanding of numbers and operation in number-related problem-solving context. Making estimation is also a part of this skill. In overall, according to NCTM (2000), students' number sense can be determined by investigating students' (1) understanding of numbers (meaning of numbers); (2) understanding of ways to represent numbers; (3) understanding of the relationship among numbers; (4) understanding of number system; (5) understanding of the meaning of numbers operations; (6) understanding of the relationship among operations; (7) ability in compute fluently; and (8) ability in making reasonable estimation (see Table 1).

A slightly different perspective proposed by Fosnot & Dolk (20010. They argue that once students acquire senses of numbers, they could decide computational strategies that fit with a problem being dealt with among other strategies in their repository (Fosnot & Dolk, 2001). In deciding the strategies, they consider the characteristics of the problems (the context), the involved numbers and the operations, the relationships within the numbers and the operations, and the relationships between numbers and operations. Here, students' number sense is built upon a complex system of cognitive development about numbers, operations, and how they are related. That comprehensive understanding leads to the development of big ideas, strategies, and models of thinking toward numbers and operations which help them to deal with computational problems fluently and flexibly. Moreover, Fosnot & Dolk (2001) assert that students who have a well-developed number sense do not rely on a specific static rote algorithm or procedure in dealing with computational problems. Instead, they rely on their mental understanding about the problems (mental mathematics) to emerge various counting strategies and then intelligently decide the strategy among the emergent strategies that are considered to be the most efficient and fit with the problems. They even can provide multiple perspectives or approaches of solutions for a single computational problem.

Fosnot & Dolk (2001) claims that some big ideas that are crucial in the early development of students' numbers sense. In the early development students' number sense, students should be supported to develop (1) their understanding of numbers (including the idea of one-to-one correspondence when counting, cardinality, magnitude, hierarchical inclusion, compensation, and part/whole relationship), (2) their understanding of number system (including the idea of unitizing, ten-based unitizing, and place value), (3) their understanding of basic facts of addition and subtraction, such as ten-based combination, commutative property, doubling , swapping, constant difference, and cancelling. Those big ideas of early number sense can be utilized as the basis to clarify the extent of students' sense of numbers.

Another researcher, such as Andrews & Sayers (2014), formulates elements building students' foundational number sense. The elements comprise the following notions, such as (1) number cognition, its vocabulary and its meaning; (2) systemic counting, including the notion of ordinality and cardinality; (3) the relationships between numbers and quantity (a number represents a certain quantity); (4) the magnitudes of numbers and the comparisons between those magnitudes; (5) making estimation of numbers; (6) performing simple arithmetic operations, such as addition and subtraction; and (7) number patterns. Those notions can be considered as indicators once assessing students' early number sense.

Meanwhile, according to Ali (2014), students with strong number sense are indicated by proficiency in (1) using numbers in flexible ways when adding, subtracting, multiplying or dividing; (2) using benchmarks to make mathematical judgments; (3) making mental calculations and reasonable estimations; (4) making predictions; (5) understanding numerical relationships between mathematical concepts, facts and skills; and (6) recognizing unreasonable answers.

Both Jordan, Fuchs, & Dyson (2014) and Jordan & Dyson (2016) state that students' early number sense can be indicated through their understanding on the three core areas, such as the notions of numbers, number relations, and number operations. The notions of numbers include the concept of subitization, cardinality, and the symbolism of numbers. Meanwhile, the notion of number relations comprises students' understanding of the magnitude of numbers and conceptual structure of numbers (mental structure of numbers). Understanding basic facts of addition and subtraction and the ability to decompose numbers in various forms are two main notions in the early development of number sense relating to early number operation proficiency.

Meantime, Putrawangsa & Hasanah (2018) regard number sense as the ability to reason with numbers. They define number sense as the ability to utilize logical, critical and creative reasoning and understanding of numbers and operations in dealing with problems relating to numbers flexibly, effectively, efficiently, and practically. A student who has a good number sense has adequate understanding and skillful in the aspect's numbers, number operations and calculations. In the aspect of numbers, students are required to grasp the meaning of numbers (their values), the number symbolism, and the relationship among numbers in a number system. Meantime, in the aspect of number operations, students should have a mental notion of the meaning of operations, the impact of the operations, and the relationship among the operations. Meanwhile, in the aspect of calculation, students who have a good sense of number can emerge various counting strategies which based on their understanding of the numbers and operations and reasonably select one of those counting strategies that is considered more effective, efficient and practical in solving computational problems.

It is identified that not all researchers obviously relate the notion of number sense to the ability to do mathematical judgments, such as making estimation or identifying error in calculation. Only NCTM (2000) and Ali (2014) stress clearly that the ability in doing mathematical judgments is a part of number sense. NCTM (2000) describes the notion as ability in making reasonable estimation, meanwhile Ali (2014) states it as the ability in using benchmarks to make mathematical judgments and recognize unreasonable answers.

Although not stated obviously, some researchers incorporated the mathematics judgment into the ability in orchestrating numbers flexibly once dealing with number-related problems. Fosnot & Dolk (2001) and Putrawangsa & Hasanah (2018), for example, incorporate the notion once students can see various strategies in dealing with number-related problems and intelligently decide one strategy among the emergent strategies that are considered to be the most efficient and fit with the problems. The selection of the best strategy requires the students to use their mathematical judgment by considering their understanding of numbers, operations and the context of the problems.

Although the researchers regard the notion of number sense in many different ways, they all consider the notion of number sense a comprehensive faculty in dealing with numbers and number operations in computational situation or problems (see Table 1).

To enhance students' sense of numbers they should have: (1) an appropriate understanding of numbers and how they are related one to another, (2) an understanding the meaning of operations and how they are related and impact numbers or other operations, (3) a capability performing computation by utilizing their understanding of the numbers and the operations fluently and flexibly in dealing with number-related problems including making reasonable estimation, and (4) a faculty in making appropriate judgment of calculation including identifying potential error of a computation and making estimation.

It implies that assessing students' number sense means assessing students' understanding on the four aspects, such as: (1) assessing students' conceptual understanding about numbers and how they are related one to another, (2) assessing students' conceptual understanding about operations and how they are related and impact numbers or other operations, and (3) assessing students' proficiency in performing calculation correctly and flexibly in dealing with computational problems, including making reasonable estimation, and (4) assessing students' faculty in making appropriate judgment of calculation including identifying potential error of a computation and making estimation.

Table 1: Experts' different views toward the elements of number sense.

NCTM (2000)	Fosnot & Dolk (2001)	Andrews & Sayers (2014)
Students who own number sense, they: 1. Understand numbers (the meaning of numbers); 2. Understand ways to representing numbers; 3. Understand the relationship between two numbers or more; 4. Understand the number system; 5. Understand the meaning of numbers operations; 6. Understand the relationship between operations; 7. Afford to compute fluently; 8. Afford to make a reasonable estimation.	Three component constructing students' number sense: 1. Sense of Number They grasp the idea of one-to-one correspondence once counting, the cardinality and the magnitude of numbers, hierarchical inclusion, part/whole relationship, and the notion of compensation. 2. Number System They comprehend with the idea of unitizing and place value system. 3. Addition and Subtraction Facts During computation, they are proficient in utilizing ten-based combination, commutative property, doubling, swapping, constant difference, and canceling out.	Elements of students' foundational number sense: 1. Involving number cognition, its vocabulary and its meaning; 2. Incorporating systemic counting, including the notion of ordinarily and cardinality; 3. Aware of the relationships between numbers and quantity (a number represent a certain quantity); 4. Aware of the magnitudes of numbers and the comparisons between those magnitudes. 5. Aware and making estimation of numbers; 6. Being able to perform simple arithmetic operations, such as addition and subtraction. 7. Aware of number patterns.
Ali (2014)	**Jordan, Fuchs, & Dyson (2014) & Jordan & Dyson (2016)**	**Putrawangsa & Hasanah (2018)**
Students who own appropriate number sense are proficient in: 1. Using numbers in flexible ways when adding, subtracting, multiplying or dividing; 2. Using benchmarks to make mathematical judgments; 3. Making mental calculations and reasonable estimations; 4. Making predictions; 5. Understanding numerical relationships between mathematical concepts, facts and skills; and 6. Recognizing unreasonable answers.	Early development of number sense involving 1. Mastering the notions of numbers, such as the idea of subitization, cardinality, and the number symbolism. 2. Mastering the notion of number relations, such as the notion of the magnitude of numbers, and the conceptual structure of numbers (mental structure of numbers). 3. Mastering early number operation, such as understanding the basic facts of addition and subtraction and being deft in decomposing numbers in various forms.	Students' number sense comprises: 1. Understanding of numbers, such as grasping the meaning of numbers (their values), the number symbolism, and the relationship among numbers in a number system. 2.Understanding of number operations, such as understanding the meaning of basic operations, the impact of the operations, and the relationship among the operations. 3.Fluency in calculation, including being able to emerge various counting strategies and selecting one of those counting strategies that is considered more effective, efficient and practical in solving the computational problems.

3.4 Considerations in Assessing Number Sense

Number sense is developed through multilevel of cognitive development which is in line with the human cognitive development of numbers and operations. In the human cognitive development of numbers, the natural number system is the basis for more other advance number systems such as integers, fractions, real, and complex numbers. Meanwhile,

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addition is regarded as the basis for other mathematical operations, such as subtraction is considered as the inverse mathematical operation of addition, and multiplication is defined as repeated additions. More advanced mathematical operations are than developed from those basic mathematical operations, such as division and exponentiation.

Regarding the multilevel of human cognitive development of numbers and operations, students' number senses are also developed according to such a development. For instance, making sense of discrete numbers is considered easier for students to grasp comparing with making sense of numbers in fractions. Having a sense of a mathematical operation, such as addition, is required a lower cognitive load comparing with other mathematical operations, such as multiplication or division. Therefore, making sense of addition is simpler to conceive than multiplication or division. Here, it can be concluded that students' number sense is not a static process, but it is a developmental process which is developed over the time as the result of life experiences, including educational experiences.

Hence, assessing students' number sense requires to consider the level of students' cognitive development about numbers and operations. It implies that assessing early grade students' number sense may somewhat difference with those higher-grade students regarding content and method of assessment.

Regarding the multilevel of human cognitive development of numbers and operations, it exaggerates to claim that a single method or instrument of number sense assessment is relevance for the whole context of number sense assessment. For example, an instrument for assessing early graders' number sense may be not sufficient to be used to assess higher graders, or instrument of assessment involving fractions will not be apt to be used for students' who have not learned yet about fractions.

Moreover, the purpose of assessment leads to several different approaches in assessing number sense. At least, there are three different purposes for assessing students' number sense, such as screening, diagnosing, and monitoring (Roberts, 2013). Assessing number sense for screening purpose is utilized once the assessment is administrated to identify students' risks in mathematics early or to assess foundational core mathematics instructions. Meanwhile, if the assessment intends to identify students' strengths or weaknesses, to develop instructional interventions, or to categorize students, the purpose of the assessment is to diagnostic students' number sense. The last, assessment for monitoring is administrate donce the purpose is to evaluate the efficacy of instructional interventions or to take quick decisions regarding adjustment of instructional interventions.

Hence, designing and developing instrument for assessing students' number sense regards at least three considerations, such as (1) the level of students' cognitive development (whether assessing pre-school students, early grade students, high grade students, etc.), (2) the mathematical scope and focus of the assessment (whether focusing on a specific or holistic operational skill, such as addition, subtraction, multiplication, etc. on integers or fractions), and (3) the purpose of the assessment (whether assessing for screening, diagnosing, or monitoring).

4 CONCLUSIONS

The way the concepts of number sense perceiving among scientists is slightly different. While cognitive psychologists consider number sense as the ability to deal with non-symbolic ideas of numbers, educators tend to regard number sense as the faculty in dealing with both symbolic and non-symbolic ideas of numbers. However, both psychologists and educators agree that students' number sense, both symbolic and non-symbolic ideas, influence students' development of advanced concepts of mathematics and students' opportunity in the future career.

Regarding the impact of students' sense of numbers, assessing students' number sense is crucial to identify students' early risk in mathematics and to prevent them from failure both in advance mathematics and in their future career.

Assessing students' number sense means assessing students' understanding on the four aspects, such as (1) assessing students' conceptual understanding about numbers and how they are related one to another, (2) assessing students' conceptual understanding about operations and how they are related and impact numbers or other operations, and (3) assessing students' proficiency in performing calculation correctly and flexibly in dealing with computational problems, including making reasonable estimation, and (4) assessing students' faculty in making appropriate judgment of calculation including identifying potential error of a computation and making estimation.

There are at least three points to be considered once designing and developing instrument for assessing students' number sense, such as (1) the level of students' cognitive development (whether assessing pre-school students, early grade students, high grade students, etc.), (2) the mathematical scope and focus of the assessment (whether focusing on a specific or holistic operational skill, such as addition, subtraction, multiplication, etc. on integers or fractions), and (3) the purpose of the assessment (whether assessing for screening, diagnosing, or monitoring).

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