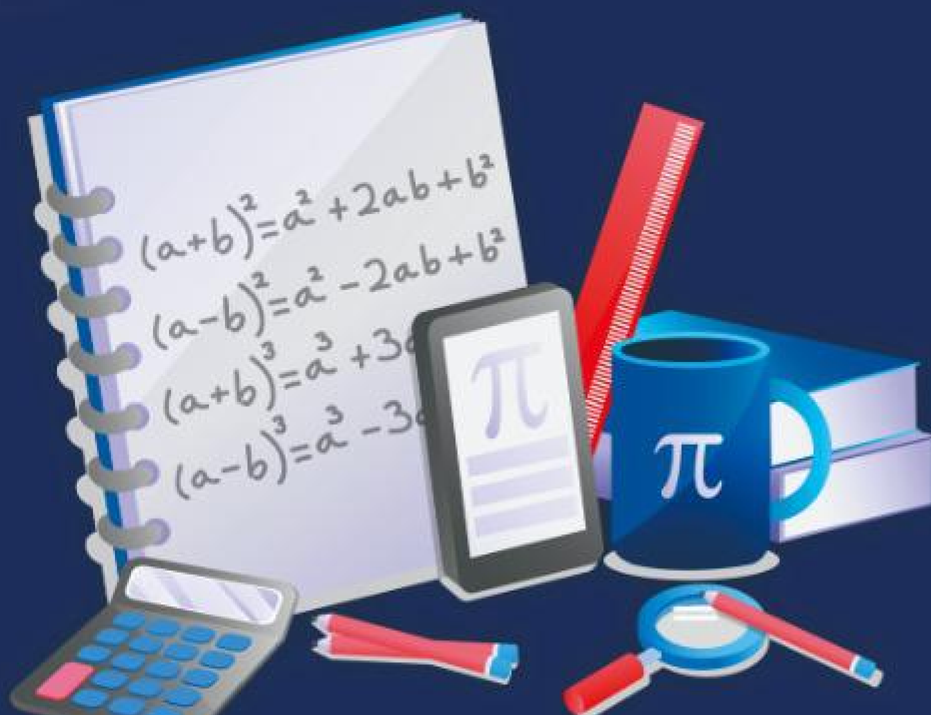
Editor: Dr. Yek Amin Azis, M.Pd.
Husnawadi, M.A. TESOL
MATHEMATICS for Madrasah Tsanawiyah (MTs) & Aliyah (MA)

Learning Materials
Development Three Languages
(Indonesia, Arab and English)

MATHEMATICS

for Madrasah Tsanawiyah (MTs) & Aliyah (MA)



Authors:
Parhaini Andriani
Alkusaeri
Lalu Sucipto
Mauludin
Kristayulita
Erpin Evendi
Nurhardiani
Samsul Irpan
Sofyan Mahfudy
Kiki Riska Ayu Kurniawati
Ahmad Nasrullah
M. Syawahid
Indira Putri Kinasih
Hesi Kumalasari

Editor:
Dr. Yek Amin Azis, M.Pd.
Husnawadi, M.A. TESOL

Learning Materials Development
Three Languages: Indonesia, Arab, & English

MATHEMATICS

for
Madrasah Tsanawiyah (MTs) & Aliyah (MA)

IMPLEMENTATING COOPERATION UIN MATARAM AND USTY

Authors:

Parhaini Andriani
Alkusaeri
Lalu Sucipto
Mauludin
Kristayulita
Erpin Evendi
Nurhardiani
Samsul Irpan
Sofyan Mahfudy
Kiki Riska Ayu Kurniawati
Ahmad Nasrullah
M. Syawahid
Indira Putri Kinasih
Hesi Kumalasari

Editor: Dr. Yek Amin Azis, M.Pd.
Husnawadi, M.A. TESOL

Mathematics for Madrasah Tsanawiyah (MTs) & Aliyah (MA)

Penulis	: Parhaini Andriani Alkusaeri Lalu Sucipto Mauludin Kristayulita Erpin Evendi Nurhardiani Samsul Irpan Sofyan Mahfudy Kiki Riska Ayu Kurniawati Ahmad Nasrullah M. Syawahid Indira Putri Kinasih Hesi Kumalasari
Editor	: Dr. Yek Amin Azis, M.Pd. Husnawadi, M.A. TESOL
Desain Cover	: Sanabil Creative

ISBN : 978-623-317-332-2
Cetakan 1 : Agustus 2022

Penerbit:
Sanabil
Jl. Kerajinan 1 Blok C/13 Mataram
Telp. 0370- 7505946, Mobile: 081-805311362
Email: sanabilpublishing@gmail.com
www.sanabilpublishing.com

Kerjasama dengan :
Universitas Islam Negeri (UIN) Mataram
UPT. Pengembangan Bahasa (UPB)
Pendidikan Bahasa Arab (PBA)
Tadris Matematika (TMTK)
Tadris Bahasa Inggris (TBI)
Jalan Gajah Mada No.100 Jempong Baru-Mataram
Website: www.uinmataram.ac.id

University of Science & Technology, Yemen (USTY)

PREFACE and INTRODUCTION BY
REKTOR UIN MATARAM

Praise be to Allah and gratitude should always be poured out in His presence enabling the completion of the book "Development of Three Language Teaching Materials, Mathematics Teaching Materials for MTs and MA" as a part of the implementation of the cooperation between the State Islamic University (UIN) Mataram and the University of Science and Technology, Yemen (USTY) which was further coordinated by UPT. P2B (The Language Development Unit of UIN Mataram, which finally presents to the readers according to the predetermined time limit.

In the development process, UPT. Language Development (UPB) UIN Mataram as the coordinator involves competent lecturers from Tadris Mathematics (TMTK), Tadris English (TBI), Arabic Language Education (PBA), and University of Science and Technology, Yemen (USTY) in the authorship of this book.

We warmly welcome the publication of this book, and we hope that the existence of this book can be used as a reference for teaching materials for Mathematics courses to complete references for teaching materials for anyone who needs it.

Finally, to all parties, especially the writing team and publishers who have played an active role in the preparation of this book, we thank you very much. Hopefully all of this can provide the maximum benefit for the implementation of various forms of innovative teaching and have a spirit of change in the desired direction, value worship in the sight of Allah, and have a positive impact on the environment. Amin.

UIN Mataram, 29 August 2022

Rector,

Prof. Dr. H. Masnun Tahir, M.Ag.

TABLE OF CONTENTS

CHAPTER I
NUMBER

This chapter discusses the real number system consisting of natural numbers, whole numbers, fractions, integers, rational numbers, and real numbers.

A. Hindu-Arabic Numeral System

The number system we use today is called the Hindu-Arabic number system, which consists of digits containing ten symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. This system utilizes base 10, where the increase in place value is increased by the power of ten. In addition, digits are employed as combinations to represent all possible numbers. Thus, this system is positional, meaning that the symbol position affects the symbol value in the number. For instance, the symbol 3 position in the number 435,681 provides a much greater value than symbol 8.

This number system was originally developed by Indian scientists around the third century BC when Brahmi numerals were applied. The Brahmi number was more complicated than the number system we use today. The Brahmi numerals have different symbols for the numbers 1 through 9, as well as different symbols for 10, 100, 1000,…, also for 20, 30, 40,…, and others for 200, 300, 400, …, 900.

Moreover, the development of the positional decimal system dates back to Hindu mathematicians during the Gupta period. Around the year 500, an astronomer named Aryabhata used the word kha ("emptiness") to denote "zero" in a table of numerals. Subsequently, a Hindu scientist, Brahmasphuta Siddhanta, in the 7th century offered a relatively advanced understanding of the role of mathematics from scratch. This Indian development was picked up in Islamic mathematics in the 8th century.

B. Whole Number

A number is an idea or abstraction that represents a quantity. Number symbols are called numbers.

There are three main functions of numbers:

1. Determine the number of elements in the finite set, referring to cardinal numbers.
2. Determine the order, referring to the ordinal numbers.

3. Implemented as identities, such as telephone, account, and population identification numbers. This function refers to identification numbers.

The concept of cardinal numbers must first be understood before discussing the more comprehensive number system. This cardinal number is usually known as the natural number.

Consider the two sets in Figure 1 below!

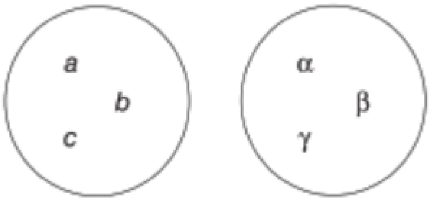


Figure 1. Two sets with different elements.

The first and second sets have distinct elements. For example, the first set consists of the letters {a, b, c} and the second set consists of {α, , }. Although these two sets are formed from different elements, they are equivalent due to their exact attributes associated with the number 3.

The symbol **n(A)** represents the number of elements **in the finite set A**.

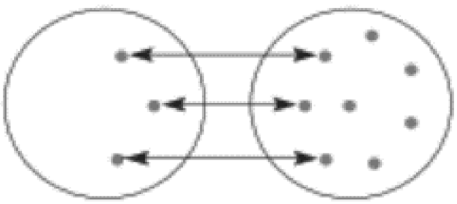
n({a,b,c})=3 because {a,b,c}~{1,2,3}

n({a,b,c,⋯,z})=26 because {a,b,c,⋯,z}~{1,2,3,⋯,26}

Definition of Whole Numbers Order

e.g. a=n(A) and b=n(B). a less than b is written "a<b" or b greater than a is written "b> an" if the set A is equivalent to a proper subset of the set B.

For instance, in Figure 2, set A has 3 elements, and set B has 8 elements. Therefore, set A is a proper subset of set B. This concept can be implemented to state that 3 < 8 (read 3 is less than 8).



Set A Set B

Figure 2. Correspondence between set A and set B

The inequality symbol can also be combined with the equality symbol expressed in symbols: a≤b (read a is less than or equal to b) and b≤a (read b is more than or equal to a).

A number line can also express the order of whole numbers, as shown in Figure 4. The number line shows the order of numbers from smallest to largest, meaning the greater the number to the right, and vice versa. Because the number 3 is further to the left than the number 8, then 3 is smaller than 8 (written a<b)

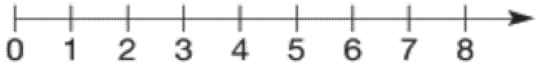


Figure 4. Number line

C. Algorithm for whole number operations

An algorithm is a procedure that applies systematic steps to find an answer, usually used for calculations.

1. Addition algorithm

A commonly written algorithm for addition involves two main procedures: (1) adding a digit (thus using basic facts) and (2) bringing (regrouping or swapping).

The place value method can be applied in an extended form, as in the following example:

$$\begin{aligned}134 + 325 &= (1 \cdot 10^2 + 3 \cdot 10 + 4) + (3 \cdot 10^2 + 2 \cdot 10 + 5)\\ &= (1 \cdot 10^2 + 3 \cdot 10^2) + (3 \cdot 10 + 2 \cdot 10) + (4 + 5)\\ &= (1 + 3)10^2 + (3 + 2)10 + (4 + 5)\\ &= 400 + 50 + 9\\ &= 459\end{aligned}$$

2. Subtraction algorithm

The general algorithm for subtraction involves two main procedures: (1) subtraction of numbers determined by the addition fact table and (2) swapping or regrouping (the opposite of the addition process). This exchange procedure usually uses the term "borrowing."

For example, to operate 323 – 64, the cascade reduction procedure is used, as shown in Figure 5 below.

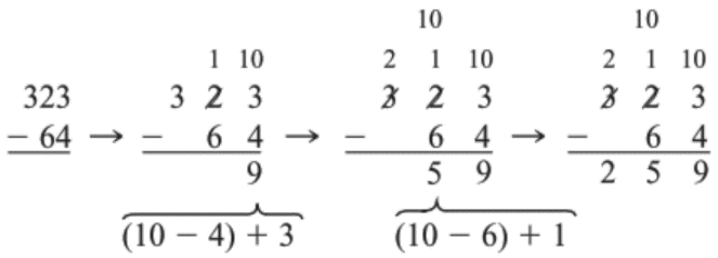


Figure 5 Example of the procedure for subtracting consecutive numbers

3. Multiplication Algorithm

Standard multiplication algorithms involve multiplication facts, distributions, and a thorough understanding of place values.

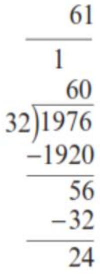
The following is an example procedure for multiplying 34 x 12

$$
\begin{aligned}
34 \times 12 \quad &= 34(10+12) \\
&= 34 \cdot 10 + 34 \cdot 2 \\
&= (30+4) \cdot 10 + (30+4) \cdot 2 \\
&= 30 \cdot 10 + 4 \cdot 10 + 30 \cdot 2 + 4 \cdot 2 \\
&= 300 + 40 + 60 + 8 \\
&= 300 + 100 + 8 \\
&= 408
\end{aligned}
$$

4. Division algorithm

The division operation involves a long division algorithm involving one or possibly two digits of the divisor. The main idea of the extended division algorithm is a division algorithm that reads, "if a and b are any whole numbers with b 0, then there are unique whole numbers q and r such that a = bq + r, where 0≤r<b".

The following is an example of the 1976 division procedure: 32.



Step 1: what is approximately 1976 divided by 32? Suggestion: 60

Step 2: Operate 1976 – (60 x 32) = 1976 – 1920 = 56

Step 3: what is approximately 56 divided by 32? Suspect: 1

Step 4: Operate 56 – 32 = 24

Based on the above calculations, a statement based on the division algorithm can be created as follows:

$$32 \cdot 61 + 24 = 1952 + 24 = 1976$$

5. Operation Order

Operations on numbers include addition, subtraction, multiplication, division, and exponents. Confusion often occurs when facing more than one operation on a particular mathematical statement. For example, when operating 3+4×5, which one was operated first? If 3 + 4 is solved first, then we get 7×5=35. However, if 4×5 is operated first, then 20+3=23 is obtained. These two procedures give different results. To eliminate confusion in performing procedures that involve more than one operation, mathematicians agree on a sequence of operations.

Operation Orders on Numbers
Brackets, Exponents, Multiplication and Division, Addition and Subtraction.
PEMDAS (Parentheses, Exponent, Multiplication, Division, Addition, and Subtraction)

Example:

Implement the ordering property of number operations to solve the following statements:

a. $5^3 - 4 \cdot (1+2)^2$

b. $11 - 4 \div 2 \cdot 5 + 3$

Solution:

a. $5^3 - 4 \cdot (1+2)^2 = 5^3 - 4 \cdot 3^2$

$= 125 - 4 \cdot 9$

$= 125 - 36$

$= 89$

b. $11 - 4 \div 2 \cdot 5 + 3 \qquad = 11 - 2 \cdot 5 + 3$

$= 11 - 10 + 3 = 4$

D. Fractions

Definition

A fraction is a number that can be represented as an ordered pair of whole numbers a/b, where b≠0. In a/b, a is the numerator, and b is the denominator.

Two fractions that represent the same number are called equivalent fractions. For example, 2/8 and 1/4 represent the same lot as Figure 6 below.

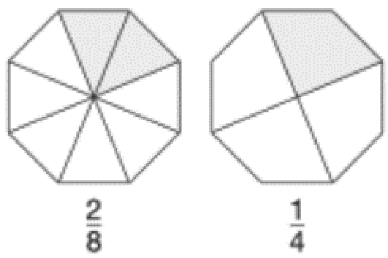


Figure 6. Representation of two equivalent fractions

Definition of fractions order

e.g., a/c and b/c be any two fractions. a/c<b/cif and only if a<b.

Example:

3/7<4/7 if and only if 3<4

Theorem

e.g., a/b and c/d be any two fractions. a/b<c/dif and only if ad<bc.

Example:

9/11<7/8 if and only if 9·8<11·7 or 72<77

Theorem

Suppose a/b and c/d are any two fractions where a/b<c/d then applies

$a/b<(a+c)/(b+d)<c/d$

Decimal

Decimals are applied to represent fractions in base ten place value notation. The decimals we have studied so far are terminating decimals because they can be represented using any number of non-zero digits to the right of the decimal point.

Example:

$$\frac{43}{1250} = \frac{43}{2 \cdot 5^4} = \frac{43 \cdot 2^3}{2^4 \cdot 5^4} = \frac{344}{10.000} = 0{,}0344$$

Theorem

Suppose a/b is a fraction in its simplest form. Then a/b has an infinitely repeating decimal representation (not terminate) if and only if b has a prime factor of 2 or 5.

Example:

Express 0,(34) as a fraction

Solution:

n = 0,(34) =0,343434….

Then 100n = 34,(34) =34,343434….

$100n = 34{,}343434 \ldots$

$n = 0{,}343434 \ldots$ -

$99n = 34$

$$n = \frac{34}{99}$$

Ratio

The concept of ratio occurs in many places in mathematics and everyday life.

A ratio is an ordered pair of numbers written a: b with b 0. The ratio a and b can also be written as a/b.

Definition of equality in the ratio

Suppose a/b and c/d are any two ratios. Then a/b=c/dif and only if ad = bc

A proportion is a statement of two equal ratios. Concepts are employed to solve problems related to ratios.

Example:

Tadika Puri Kindergarten ordered 3 boxes of chocolate milk for every 7 children. If there are 581 students in the school, how much chocolate milk should be ordered?

Solution:

$$\frac{3}{7} = \frac{n}{581}$$

Using the cross-product property of the ratio, we get

$$7n = 3 \cdot 581$$

$$n = \frac{3 \cdot 581}{7} = 249$$

Tadika Puri Kindergarten had to order 249 boxes of chocolate milk.

Percent

The word percent comes from Latin, which means "per hundred." So 25 percent means 25 per hundred, 25/100, or 0.25, which is "25%".

Since percents can be expressed as fractions with a denominator of 100, problems involving percent involve three pieces of information, namely a percentage, p, and two numbers, a and n, which are the numerator and denominator of a fraction. The relationship between the three is expressed in the following proportions:

$$\frac{p}{100} = \frac{a}{n}$$

Example:

A shirt has a price of Rp. 150,000.00 and it gets a 30% discount. How much do we have to pay to buy clothes?

Solution:

For example, p = 30 and n = 150,000

Therefore

$$\frac{30}{100} = \frac{a}{150.000}$$

$$a = \frac{30}{100} \times 150.000 = 45.000$$

Discount on the price of clothes= Rp. 45,000

The money to be paid to buy clothes = Rp. 150,000 - Rp. 45,000 = Rp. 105,000.

E. Integer

There are situations in mathematics where negative numbers are required. For example, subtraction 4 – 7 has no answer on whole numbers. For this reason, it requires an integer number.

Definition

Integer is the set I={…,-3,-2,-1,0,1,2,3,…}. The numbers -1, -2, -3, … are called negative integers. The numbers 1, 2, 3, … are called positive integers. Zero is neither a negative nor a positive integer.

The properties of addition to integers

For instance, a and b be any integer

1. The closed property of addition.

   a + b is an integer

2. Commutative property of addition

a+b=b+a

3. The associative property of addition

a+(b+c)=(a+b)+c

4. The identity property of addition

0 is a unique integer such that a+0=0+a=a, which applies to all a.

5. Additive inverse property of addition

A unique integer, written -a, for every integer a, such that a+(-a)=0. The integer -a is called the additive inverse of a.

Definition of subtraction on integers

If a and b are any integers, then a-b=a+(-b)

Multiplication on integers

For example, a and b be any integer

1. Multiply by 0. a·0=0·a=0
2. Multiply two positive integers.
   If a is positive and b is positive, then a·b=ab
3. Multiplying positive and negative integers
   If a is positive and b is negative, then a·(-b)=-ab
   If a is negative and b is positive, then (-a)·b=-ab
4. Multiply two negative integers
   If a is negative and b is negative, then (-a)·(-b)=ab

Multiplication properties of integers

For instance, a and b be any integer

1. The closed property of multiplication.
   a·b is an integer
2. The commutative property of multiplication
   a·b=b·a
3. The associative property of multiplication
   a·(b·c)=(a·b)·c
4. The identity property of multiplication
   1 is a unique integer such that a·1=1·a=a, which applies to all a.
5. The inverse property of multiplication

For every integer a, a unique number is written 1/a, such that $a \cdot 1/a=1$. The number 1/a is called the product inverse of a.

6. The distributive property of addition and multiplication

$a\cdot(b+c)=(a\cdot b)+(a\cdot c)=ab+ac$

F. Rational numbers

In solving various mathematical problems, it is necessary to use reciprocal and opposite numbers. The reciprocal property applies to fractions, and the opposite applies to integers. Therefore we need a new number called the rational number, which combines the properties of fractions and integers.

The rational numbers are the set $Q=\{a/b \mid a \text{ and } b \text{ are integers}, b\neq 0\}$

Examples of rational numbers 4/3, (-5)/7,4/(-9),0/1,(-7)/(-9),-3 1/4,(-13)/4 and 2 1 /3=7/3

Operations on rational numbers are the same as operations on fractions and integers.

G. Real Numbers

Each repeating decimal can be expressed as a rational number a/b where a and b are integers. So a non-repeating decimal number is not a rational number. Solving equations plays an essential role in mathematics. Therefore we need a number system that allows us to solve various equations. So we need a new number system that is wider called the natural number system.

Definition

The set of real numbers, denoted R, is the set of all numbers with an infinite decimal representation.

Real numbers include all rational numbers and irrational numbers. A rational number is the set of all numbers with infinite repeating decimal representations, positive, negative, or zero.

An irrational number is the set of all numbers that have infinite non-repeating decimal representations.

Examples of irrational numbers: $\sqrt{2}$, $\pi = 3{,}14159$ ...

EXERCISE

1. If two numbers produce an even number and their addition produces an odd number, what can be concluded from the two numbers?
2. Find three fractions greater than $\frac{2}{5}$ and less than $\frac{3}{7}$.
3. An auditorium contains 315 seats and $\frac{7}{9}$ of which are occupied. How many seats are still empty?
4. The price of 1 kg of rice in 2012 in West Nusa Tenggara Province was Rp. 7,700. In 2022, it rises to Rp. 10,750 per kg. What is the percentage increase in the price of 1 kg over 10 years?
5. Express the following decimal form as a fraction:

a. $0,\overline{123}$

b. $0{,}001\overline{78}$

6. A farmer calculates that for every 100 corn seeds planted, he will get a yield of 84 corn plants. If the farmer wants a yield of 7200 corn plants, how many corn seeds should he plant?
7. Ana states that for integers a and b, (a+b)(a+b)=a^2+b^2. Is the statement always true, never true, or sometimes true? Explain your reasons.
8. Arrange the following pairs of rational numbers:

a. $-\frac{3}{7}, \frac{5}{2}$

b. $-\frac{5}{7}, -\frac{3}{4}$

9. Determine whether the following decimals represent rational or irrational numbers

a. 0,273

b. 2,718281828…

c. $-15{,}00\overline{15}$

10. The number is an example of an irrational number. Practically, $\frac{22}{7}$ is used to refer to the π value. Is it true that $=\frac{22}{7}$? Explain your reasons.

CHAPTER II

A SET

This chapter discusses the concept of a set consisting of its definition, how to present, types, and operations.

A. Definition

A set is a collection of objects which is defined clearly. It is also defined as a collection of specific objects. In this case, specific is distinguished by contrast between one another, and it also indicates that the set involves certain regulations known as membership requirements. For example, if we ask a child who cannot count to collect red flowers among many colorful flowers, and he can do it, he shows that he understands the meaning of membership conditions. So, it can be said that the set is a collection of objects that meet the membership requirements (to become members of the set).

To declare a set, one must follow the rules for presenting the set, but it is necessary to know the symbols for the formation of the set:

- Denoted by a capital letter
- flanked by two curly braces
- Separated by a comma (,) on each set member.
- In general, it is shown as **A = {.........}**

Membership in a set is shown in the following way:

$x \in B$: *x* is the member of set *A*;

$x \notin B$: *x* is not the member of set *A*.

B. How to Present a Set

According to its writing symbol, practically, there are several ways of presenting a set, namely:

1. Enumeration

It is the presentation of a set where the members are shown one by one (counted).

e.g.: The set of the first four natural numbers: $A = \{1, 2, 3, 4\}$.

2. A Set Builder Notation

The general form of the Set Builder Notation is:

A : { x | a must requirement by x }

e.g.:

A is the positive integer smaller than 5

A = { x | x is the positive integer smaller than 5}, or

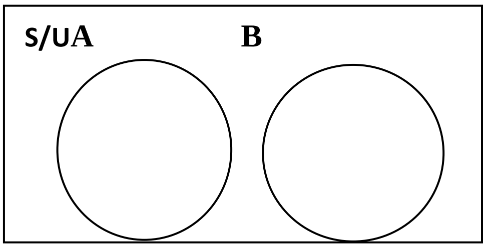
A = { x | $x \in P$, x< 5 } which is equivalent to A = {1, 2, 3, 4}

3. Venn Diagram

A Venn diagram is one way to present a set by describing it using certain rules to make the set looks more practical. It is helpful for presenting a set if it is more complex and contains relationships based on set operation rules. These are several ways to present a set using this diagram:

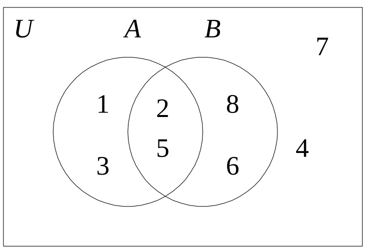
- Members of the universe conversation are in a square
- Place the symbol S or U in the upper left corner of the square
- Symbolized by capital letters to indicate a certain set
- Using a circular line to delimit a certain set

Picture of Venn Diagram



Example 5.

If U = {1, 2, …, 7, 8}, A = {1, 2, 3, 5} and B = {2, 5, 6, 8}. The diagram can be:



4. Standard Symbols

This applies to the set which is based on agreement that become joint understanding used in its show. Here are the symbols:

N = a set of natural numbers = { 1, 2, ... }

Z = a se of integers = { ..., -2, -1, 0, 1, 2, ... }

Q = a set of rational numbers

R = a set of real numbers

C = a set of complex number

S = a set of universe

C. Types of Sets

Below are the types of a set:

1. A set of cardinal

The cardinal set is the number of elements in a set. For example, the number of elements in *A* is called the cardinal of the set A

Notasi: $n(A)$ or $\lvert A \rvert$

Example:

a) B = { x | x is prime number smaller than 20 }, or B = {2, 3, 5, 7, 11, 13, 17, 19} so $\lvert B \rvert$ = 8

b) T = {kucing, a, Amir, 10, paku}, so $\lvert T \rvert$ = 5

c) A = {a, {a}, {{a}} }, so $\lvert A \rvert$ = 3

2. Empty Set

This one has no member or cardinal set = 0

Notation: ∅ or {}

Example:

a) E = { x | $x<x$ }, then $n(E) = 0$

b) P = { Indonesian who went to the moon }, then $n(P) = 0$

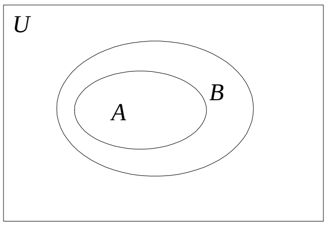
c) A = {x | x is the root of the quadratic equation $x^2 + 1 = 0$ }, $n(A) = 0$

3. The Subset

Set *A* is said to be part of set *B* if and only if element *A* is element *B*. in this case, *B* is the *superset* of *A*.

Notation: $A \subseteq B$

Showed with Venn diagram, here is it:



Example:

a. $\{1, 2, 3\} \subseteq \{1, 2, 3, 4, 5\}$

b. $\{1, 2, 3\} \subseteq \{1, 2, 3\}$

c. $\mathbf{N} \subseteq \mathbf{Z} \subseteq \mathbf{R} \subseteq \mathbf{C}$

TEOREMA 1. For any set A, the following applies:

(a) A is a subset of A itself (, $A \subseteq A$).

(b) an empty set is a subset of A ($\varnothing \subseteq A$).

(c) if $A \subseteq B$ and $B \subseteq C$, so $A \subseteq C$

- $\varnothing \subseteq A$ and $A \subseteq A$, so $\varnothing$ and A defines as an *improper subset* of A.

e.g.: $A = \{1, 2, 3\}$, so $\{1, 2, 3\}$ and $\varnothing$ is *improper subset* of A.

- $A \subseteq B$ is different from $A \subset B$

a) $A \subset B$: A is a subset of B but $A \neq B$.

A is the *proper subset* of B.

e.g.: $\{1\}$ and $\{2, 3\}$ is proper *subset* of $\{1, 2, 3\}$

b) (ii) $A \subseteq B$: is used to state A as *subset* of B allowing $A = B$.

4. A Similar Set

$A = B$ if and only if each element A is B and vice versa element B is A. It can be said $A = B$ if A is a subset of B and vice versa. If not, then $A \neq B$.

Notation: $A = B \leftrightarrow A \subseteq B$ and $B \subseteq A$

Example:

a) If $A = \{0, 1\}$ and $B = \{x \mid x(x - 1) = 0\}$, then $A = B$

b) If $A = \{3, 5, 8, 5\}$ and $B = \{5, 3, 8\}$, then $A = B$

c) If $A = \{3, 5, 8, 5\}$ and $B = \{3, 8\}$, then $A \neq B$

For three sets, A, B, and C, it applies the following axioms:

1) $A = A$, $B = B$, and $C = C$

2) If $A = B$, then $B = A$

3) If $A = B$ dan $B = C$, then $A = C$

5. An Equivalent Set

A is equivalent to *B* if and only if both cardinals are similar.

Notation: $A \sim B \leftrightarrow |A| = |B|$

Example:

If $A = \{ 1, 3, 5, 7 \}$ and $B = \{ a, b, c, d \}$, then $A \sim B$ because $|A| = |B| = 4$

6. Independent Sets

Both sets A and B are *disjoint* when they have different elements.

Notation: $A // B$

Example: if $A = \{ x \mid x \in P, x < 8 \}$ and $B = \{ 10, 20, 30, ... \}$, then $A // B$.

7. Power Set

Power set of A is the one where its elements is a subset of A including the empty set and A itself.

Notation: $P(A)$ or 2^A

If $|A| = m$, then $|P(A)| = 2^m$.

Example:

If $A = \{ 1, 2 \}$, then $P(A) = \{ \varnothing, \{ 1 \}, \{ 2 \}, \{ 1, 2 \}\}$

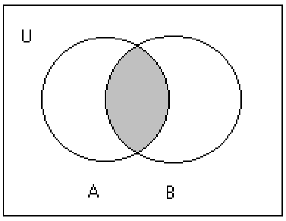
D. Operation of Sets

This section discusses the operations of sets applied to two or more, and it cannot be done on one set only. The sentence 'cannot' here does not mean absolutely cannot be done, rather it points out that the operations applied to the same two sets will not give different results. So, it can still be done but does not provide any benefit.

1. Intersection

set A intersection B is a set of members consisting of elements which is simultaneously in A nor B.

Notation: $A \cap B = \{ x | x \in A \text{ and } x \in B \}$



Example:

If A = {2, 4, 6, 8, 10} and B = {4, 10, 14, 18},

then A ∩ B = {4, 10}

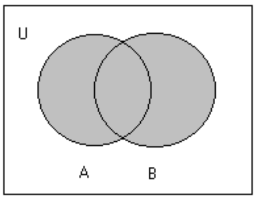
If A = { 3, 5, 9 } and B = { -2, 6 }, then A ∩ B = ∅.

Meaning: A // B

2. Union

Union of sets A and B is the one with member A or B or both.

Notation: $A \cup B = \{ x | x \in A \text{ or } x \in B \}$



Example:

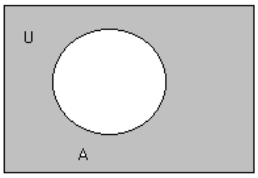
a) If A = { 2, 5, 8 } and B = { 7, 5, 22 }, then $A \cup B$ = { 2, 5, 7, 8, 22 }

b) $A \cup \varnothing = A$

3. Complement

If there is set A, then complement set A is a member of universe set, yet it is not a member of set A.

Notation: $\overline{A} = \{ x | x \in U, x \notin A \}$



Example:

If U = { 1, 2, 3, ..., 9 },

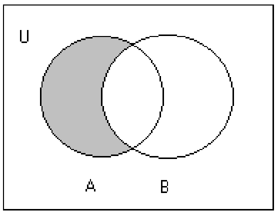
a) If A = {1, 3, 7, 9}, then $\overline{A}$ = {2, 4, 6, 8}

b) If A = { x | x/2 $\in P$, x< 9 }, then $\overline{A}$= { 1, 3, 5, 7, 9 }

4. Difference

set A difference set B is a member of set A which excluded from B and vice versa if not doing difference operation of set B – A.

Notation: $A - B = \{ x | x \in A \text{ and } x \notin B \} = A \cap \overline{B}$



Example:

a. If A = { 1, 2, 3, ..., 10 } and B = { 2, 4, 6, 8, 10 }, then $A - B$ = { 1, 3, 5, 7, 9 } and $B - A = \varnothing$

b. {1, 3, 5} – {1, 2, 3} = {5}, but {1, 2, 3} – {1, 3, 5} = {2}

5. Symmetric Difference

Symmetric Difference of sets A and B, written as A ⊕ B, is a set where the members are mixture of both sets A and B, but not as intersection member of sets A and B.

Notation: $A \oplus B = (A \cup B) - (A \cap B) = (A - B) \cup (B - A)$

Example:

IfA = { 2, 4, 6 } and B = { 2, 3, 5 }, then $A \oplus B$ = { 3, 4, 5, 6 }

6. Cartesian Product

The Cartesian product of sets A and B is an ordered pair *a* with *b*, where *a* is the member of set A while *b* is the member of set B.

Notation: $A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$

Example:

If C = { 1, 2, 3 }, and D = { a, b }, then $C \times D$ = { (1, a), (1, b), (2, a), (2, b), (3, a), (3, b) }

EXERCISE

1. Which of the following collections is a set?

a. A group of students from class IA MTs NW Kembang Kerang.

b. East Lombok Mathematics Teacher Association.

c. The set of natural numbers

d. A group of students who live far from campus.

e. Association of successful entrepreneurs.

f. The set of very large numbers

2. Write in set builder notation and enumeration as follows:

a. A set of odd numbers less than 13

b. A set of odd numbers between 2 and 12

c. A set of prime numbers less than 13

d. A set of prime numbers between 2 and 12

e. C = {3, 6, 9, 12, 15}

3. What is the cardinality of the set A = { $x \mid x$ <50, x □ Prime number}

4. Which of the following sets is an empty set?

A. A set of negative root numbers *a*

B. Association of Students Mathematics

C. Association of UIN students who do not wear headscarves

5. Determine the set power of B={1, 2, 3,4}

6. Given A = {a, b, c}, B = {a, b, d, e, g}, C = {a, b, c, d, e, f}. which of these sets contains a relationship as a subset?

7. Given A = { a, 1, b, 2, c, 3} B = { p, 4, q, 5, r, 6}, Determine A + B!

8. Determine A + B if the set:

A = A set of prime numbers less than 10

B = A set of composite numbers less than 10

9. Given sets defined as follow:

A = { x | 0 □ x □ 18, x □ when the original is divisible by 3}

B = { x | 9 □ x □ 19, x □ Prime numbers }

C = { x | -8 □ x □ 7, x □ B }

Determine:

a. A + (B □ C)

b. B + (A □ C)

c. C + (A □ B)

d. (A □ B) + (A □)

10. Determine A – B if:

A = {p, q, r, s, t}

B = {3, r, 6, s, 9, t}

CHAPTER III
MATHEMATICAL LOGIC

In this chapter discussed mathematical logic concepts consisting of: statements (propositions), statements operations, compound/pluralistic statements, tautology and contradictions and drawing the conclusions.

A. STATEMENT (PROPOSITION)

"The rooster crows., in the proposition, it has another meaning, that "crows" refers ti the rooster." An explanatory understanding is called a predicate (P) and the meaning explained is called the subject (S).

S: Rooster

***P*:**crows

If the word "it" or the explanatory function is marked = then the pattern of the proposition is written with S = P.

If there is a denial, then the proposition is formed: :

"The rooster does not crowd", and positioning pattern is : $S \neq P$.

If it is recognized that the rooster does crow, or it does not crow, it means that true the rooster crow or does not crow. .

So it is clear that the propositionv(statement), hava a right or wrong character. .

<u>The definitions of statements:</u>:

A statement is a sentence that are either right or wrong, but both of them right and wrong.

1. Open Sentences, Modifier (Variables), Constants, and Open Sentences Completion (Repetition)

a. Definition of Open Sentence

An open sentence is a sentence that contains a variable and becomes a statement if the variable is replaced by a constant in the universal set.

Example:

1) City L is a tourist area.

2) 2 + x = 8

b. Definition of Variable

A variable is a symbol to denote any member of the universal set.

Example:

1)…. + 6 = 1 (…. is a variable)

2) $x - 2 = 5$

c. Definition of Constant

Constants are symbols to indicate certain members in the universal set.

Example:

1) House…. in Lombok.

If…. replaced with Cipto then Cipto is called a constant in the set of the human universe. The truth of the statement "Cipto's house in Lombok" depends on the reality.

2) ….+ 6 = 11

If …. with 3 then the statement $3 + 6 = 11$ is false and 3 is called a constant.

3) $x - 2 = 5$

If x is replaced by 7 then the statement $7 - 2 = 5$ is true and 7 is called a constant.

4) $y + 1 < 5$

If the universal set is natural numbers and y is replaced by 1,2, and 3 then the statement is true and 1,2, and 3 are called constants.

d. The Set of Completion of an Open Sentence

Example 1:

$2x - 1 < 5$; $x \in \{0, 1, 2, 3, 4, 5\}$

The sentence becomes a true statement if x is replaced by 0, 1, and 2

So, the solution set is {0, 1, 2}.

Example 2:

$x^2 + 5x - 24 = 0$

The sentence becomes a true statement if x is replaced by -8 and 3

So, the solution set is { -8, 3 }.

From the examples above it can be concluded that:

- *Completion of an open sentence* are constants in place of variables that cause the open sentence to be a true statement.
- The set that contains all possible solutions is called the solution set.

2. Proportional Logic Symbol System

No.	Operator	Meaning in everyday language

massage	Name	Symbol	
1	negation	~	No, no
2	Conjunction	$\wedge$	And, but, even though, even though
3	Disjunction	$\vee$	Or
4	Implication	$\Rightarrow$	If then …
5	Biimplication	$\Leftrightarrow$	If and only if…then…

3. Denial or Negation of a Statement

If p is a statement then the circle is denoted as ~p or –p. If the p statement is true, then the ~p statement is false. On the other hand, if the statement p is false, then the statement ~p is true.

Example 1:

p: Azam wears a white shirt

~*p*: It's not true that Azam is wearing a white shirt, or

~*p*: Azam doesn't wear white clothes.

The truth value of the statement p depends on reality, if p is true,

Then ~p is false or vice versa.

Example 2:

p: 3 + 2 = 7 …………………………. (S)

~*p*: 3 + 2 7 ………………………….. (B)

The truth value of the negation is presented in the truth table below.

p	~*p*
B	S
S	B

B. STATEMENT OPERATION

1. Conjugation

The conjunction of two statements p and q is true only if both components are true.

The truth values of conjunctions are presented in the truth table below.

P	*q*	$p \wedge q$

B	B	B
B	S	S
S	B	S
S	S	S

Words that form conjunctions other than and are although, but, while, also, although.

2. Disjunction

If the statements p and q are connected by a conjunction or, then the statements p or q are called disjunctions.

Disjunction Truth Values and Tables

a. Inclusive disjunction of two statements p and q, i.e. $p \vee q$, i.e. $p \vee q$is true if one or both of the statements p and q are true.

b. The exclusive disjunction of two statements p and q is true only if one of the statements p and q is true.

The truth value of the disjunction is presented in the truth table below.

		Inclusive disjunction	Exclusive disjunction
P	*q*	$p \vee q$	*pq*
B	B	B	S
B	S	B	B
S	B	B	B
S	S	S	S

Determining the Truth Value of the Sentence $p \vee q$

Example:

Determine the value of x so that the sentence $x^2 - 4 = 0 \vee 1 - (-1) = 0$ is false.

Answer:

$p(x)$: $x^2 - 4 = 0$

$(x - 2)(x + 2) = 0$

$x = 2 \vee x = -2$

q: $1 - (-1) = 0$ ....(S)

Then the sentence $p(x) \vee q \equiv S$ if p(x) is false

X	p(x)	q	p(x)$\vee q$
x= ±2	B	S	B
x= ±2	S	S	S

So, so that $x^2 - 4 = 0 \vee 1 - (-1)$ is false then p≠± 2.

3. Implication

The implications of the two p . statements$\Rightarrow q$is false only if p is true and q is false.

The truth values of the implications are presented in the truth table below.

p	q	$p \Rightarrow q$
B	B	B
B	S	S
S	B	B
S	S	B

Determining the Truth Value of the Sentence p(x)$\Rightarrow q$

Example 1:

Its known that p(x): $x^2 - 1 = 0$ and q: $2\times3 = 6$

Determine x so that p(x)$\Rightarrow q$true value.

Answer:

Since q is true, then for p(x) it is true or false, the implication of p(x)$\Rightarrow q$remains true.

x	p(x)	q	p(x)$\Rightarrow q$
x= ± 1	B	B	B
x= ± 1	S	B	B

So, p(x)$\Rightarrow q$or $(x^2 - 1 = 0)\Rightarrow(2\times3 = 6)$ is true for all $x \in R$

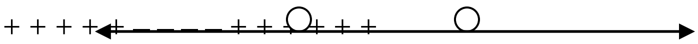
Example 2:

Determine x so that $(x^2 - 4 \leq 0)\Rightarrow(2\times3\neq6)$ value is false

Answer:

p(x): $x^2 - 4 = 0$

$(x - 2)(x + 2)\leq0$



$-2 \le x \le 2$

q: $2\times3\neq6$ (S)

Since q is false, so that the implication of $p(x)\Rightarrow q$ is false, p(x) must be true (see the truth table implication of the second row)

x	$p(x)$	q	$p(x)\Rightarrow q$
$-2\le x\le2$	B	S	S

So, the implication $(x^2-4\le0)\Rightarrow(2\times3\neq6)$ is false for $\{x \mid -2\le x\le2,\ x\in R\}$

a. Implications of the form $p(x)\Rightarrow q(x)$

$p(x)$ and q(x) are open sentences, respectively

1) Suppose that the solution set of p(x) is P and the solution set of q(x) is Q, then if $P\subset Q$ then $p(x)\Rightarrow q(x)$ is true

2) Implications in the form of $p(x)\Rightarrow q(x)$ which is always true is called a logical implication

3) how to determine the truth value of $p(x)\Rightarrow q(x)$ as follows:

(i) find the value of x that satisfies p(x) so that it satisfies q(x), or

(ii) find the solution set p(x) and q(x) like point 1

Example:

If x = 2 then $x^2 = 4$

$p(x)$: $x = 2 \rightarrow$ Solution set P = {2}

$q(x)$: $x^2 = 4$

$x=\pm2\rightarrow$ solution set Q = { –2, 2 }

because $\{2\}\subset\{-2, 2\}$ or $P\subset Q$ then $(x = 2)\Rightarrow(x^2 = 4)$ is true.

b. Logical implications

$p(x)$ logical implication of q(x) if and only if for every x satisfies p(x) also satisfies q(x).

Example :

Show with a truth table that: $(p\Rightarrow q)\Rightarrow p$ logical implication p

answer :

it must be shown that $((p\Rightarrow q)\Rightarrow p)\Rightarrow p$ is a tautology

p	*q*	*p*⇒*q*	(p⇒*q*)⇒*p*	((p⇒*q*)⇒*p*)⇒*p*
B B SS	BSBS	BSBB	BBSS	BBBB

Tautology

It can be shown that ((p⇒*q*)⇒*p*)⇒*p*always true for all possibilities (in the fifth column of the table above) so that ((p⇒*q*)⇒*p*)⇒*p*a tautology.

4. Biimplication

The biimplication of the statements p and q is written with the symbol p⇔*q*. Biimplication is also known as bi-directional or biconditional implication.

p symbol⇔*q*, be read:

a. *p*if and only if q

b. *p*necessary and sufficient conditions for q

c. *q*necessary and sufficient conditions for p

d. if p, then q, and if q, then p

e. *p*equivalent of q

Biimplication of p⇔*q*, is true if the truth values of the statements p and q are the same. If the truth values of the p and q statements are not the same, then the p biimplication statement⇔*q*wrong value.

The truth table of the p . biimplication statement⇔*q*, is shown below:

P	*Q*	*p*⇔*q*
BBSS	BSBS	BSSB

Example:

Suppose the statement p: 3 prime numbers (B) and the statement

q: 8 even numbers (B)

then p↔*q*: 3 is prime if and only if 8 is even (B)

let the statement p: 2^5 = 10 (S), and the statement

q: ^{2} log 100 = 2 (S)

then p↔*q*: 2^5 = 10 if and only if ^{2} log 100 = 2 (B)

C. MULTIPLE STATEMENTS

1. Equivalent Compound Statements

Two compound statements are said to be equivalent if they have the same truth value. Meanwhile, to find out whether the 2 compound statements are equivalent or not, it can be investigated using a truth table. Compound statement P is equivalent to a compound statement Q written with the symbol $P \equiv Q$.

Example:

Show with a truth table that:

a. $p \to q \equiv \sim p \vee q$

b. $p \to q \equiv q \Leftrightarrow p$

Answer:

Using the truth table:

p	q	$\sim p$	$p \Rightarrow q$	$\sim p \vee q$	$p \Leftrightarrow q$	$q \Leftrightarrow p$
B	B	S	B	B	B	B
B	S	S	S	S	S	S
S	B	B	B	B	S	S
S	S	B	B	B	B	B

From the table it can be seen that the truth value of $p \Rightarrow q$equal to the truth value $\sim p \vee q$, as well as the truth value $p \Leftrightarrow q$equal to $q \Leftrightarrow p$

Thus it is proven that:

$$p \Rightarrow q \equiv \sim p \vee q$$

$$p \Leftrightarrow q \equiv q \Leftrightarrow p$$

2. Negation of Compound Statements

a. Conjunction Negation and Disjunction Negation

The negation of $(p \wedge q)$ is$\sim(p \wedge q)$ or $(\sim p \vee \sim q)$

The negation of $(p \vee q)$ is$\sim(p \vee q)$ or $(\sim p \wedge \sim q)$

This can be proven by the following truth table:

Conjunction Negation Table

p	q	~p	~q	(p∧q)	~(p∧q)	(~p∨~q)
BBSS	BSBS	SSBB	SBSB	BSSS	SBBB	SBBB

p	q	~p	~q	(p∨q)	~(p∨q)	(~p∧~q)
BBSS	BSBS	SSBB	SBSB	BBBS	SSSB	SSSB

From the first table it can be seen that the truth value~(p∧q) is equal to the truth value (~p∨~q), while in the second table the truth value~(p∨q) is equal to the truth value (~p∧~q), this shows that:

~(p∧q)≡(~p∨~q)

~(p∨q)≡(~p∧q)

Example:

Today it's raining hard and there's flooding

Answer:

The negation of this statement is that it is not true today it rained heavily and there was a flood. This sentence is equivalent to today there is no heavy rain or no flooding.

b. Negation of Implication

negation of p⇒qis ~(p⇒q) or (p∧~q)

This can be shown by the following truth table:

p	q	~p	~q	p⇒q	~(p⇒q)	p∧~q
BBSS	BSBS	S	S	B	S	S
		S	B	S	B	B
		B	S	B	S	S
		B	B	B	S	S

So, from the table above it can be seen that the truth value ~(p⇒q) and p∧~q is equal, indicating that:

~(p⇒q)≡(p∧~q)

Example:

If there is an earthquake, the tide will rise

Answer:

The negation of the sentence is not true that if there is an earthquake then the tide will rise. Or there is an earthquake and the sea water is not high tide.

c. Negation of Biimplication

The negation of $(p \Leftrightarrow q)$ is $\sim(p \Leftrightarrow q)$ or $(p \wedge \sim q) \vee (q \wedge \sim p)$

The above negation can be shown in the following truth table:

p	q	$\sim p$	$\sim q$	$p \Leftrightarrow q$	$\sim(p \Leftrightarrow q)$	$p \wedge \sim q$	$q \wedge \sim p$	$(p \wedge \sim q) \vee (q \wedge \sim p)$
B	B	S	S	B	S	S	S	S
B	S	S	B	S	B	B	S	B
S	B	B	S	S	B	B	B	B
S	S	B	B	B	S	S	S	S

This shows that the negation of $p \Leftrightarrow q$is $\sim(p \Leftrightarrow q)$ or $(p \wedge \sim q) \vee (q \wedge \sim p)$

so

$$\sim(p \Leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$$

Example:

Football matches are postponed if and only if it rains heavily

Answer:

The negation of the sentence is that the football match is postponed and it is not raining heavily or it is raining heavily and the football match is postponed.

D. TAUTOLOGY AND CONTRADICTION

A tautology is a compound statement that always evaluates to true regardless of the truth value of its constituent statements.

A contradiction is a statement that is always false regardless of the truth value of the statement that formed it.

Meanwhile, if it is neither a tautology nor a contradiction, it is called a contingency.

Example 1:

Show that the following statement is a tautology $((p \Rightarrow q) \wedge p) \Rightarrow p$

Answer:

p	q	$p \Rightarrow q$	$(p \Rightarrow q) \wedge p$	$((p \Rightarrow q) \wedge p) \Rightarrow p$
B	B	B	B	B
B	S	S	S	B
S	B	B	S	B
S	S	B	S	B

From the table above, the truth value of the statement $((p \Rightarrow q) \wedge p) \Rightarrow p$always true value. So $((p \Rightarrow q) \wedge p) \Rightarrow p$is a tautology.

Example 2:

Show that the following statement is a contradiction $(\sim p \wedge q) \wedge p$

Answer:

p	q	$\sim p$	$(\sim p \wedge q)$	$(\sim p \wedge q) \wedge p$
B	B	S	S	S
B	S	S	S	S
S	B	B	B	S
S	S	B	S	S

From the table above the truth value of $(\sim p \wedge q) \wedge p$always evaluates to false so $(\sim p \wedge q) \wedge p$is a contradictory statement.

E. CONVERS, INVERS AND CONTRAPOSITIONS

From the implication of $p \Rightarrow q$Other implications can be formed, namely:

$q \Rightarrow p$is called the converse of $p \Rightarrow q$

$\sim p \Rightarrow \sim q$ is called the inverse of $p \Rightarrow q$

$\sim q \Rightarrow p$is called the contraposition $p \Rightarrow q$

p	q	$\sim p$	$\sim q$	$p \Rightarrow q$	$q \Rightarrow p$	$\sim p \Rightarrow \sim q$	$\sim q \Rightarrow \sim p$
B	B	S	S	B	B	B	B
B	S	S	B	S	B	B	S
S	B	B	S	B	S	S	B
S	S	B	B	B	B	B	B

So it can be concluded that an implication is equivalent to its contraposition.

$p \Rightarrow q \equiv \sim q \Rightarrow \sim p$

$q \Rightarrow p \equiv \sim p \Rightarrow \sim q$

Example:

When the sun rises, the rooster crows.

Answer:

The converse is that when the rooster crows, the sun rises.

The inverse is that if the sun doesn't rise, the rooster doesn't crow

The contraposition is that if the rooster doesn't crow, the sun doesn't rise.

F. WITHDRAWAL OF CONCLUSION

In order to draw a conclusion, a premise that is true is required

Then in a certain way a conclusion is drawn which is called a conclusion. Drawing conclusions from the premises is called argumentation. An argument is said to be valid if the conjunction of the premise has implications for the conclusion and is a tautology.

Some of the principles of valid conclusions include:

1. Principle of Ponen Mode

Premise 1: $p \Rightarrow q$(B)

Premise 2: p(B)

Conclusion: $\therefore q$(B)

P	Q	$p \Rightarrow q$	$(p \Rightarrow q) \wedge p$	$((p \Rightarrow q) \wedge p) \Rightarrow q$
B	B	B	B	B
B	S	S	S	B
S	B	B	S	B
S	S	B	S	B

So drawing conclusions using the modus ponen principle is a valid conclusion.

2. Tolens Mode Principle

Premise: $p \rightarrow q$(B)

Premise: ~q(B)

Conclusion: $\therefore$ ~p (B)

P	Q	~p	~q	$p \Rightarrow q$	$(p \Rightarrow q) \wedge \sim q$	$((p \Rightarrow q) \wedge \sim q) \Rightarrow \sim p$
B	B	S	S	B	S	B
B	S	S	B	S	S	B

S	B	B	S	B	S	B
S	S	B	B	B	B	B

3. Principle of Syllogism

Premise 1: $p \Rightarrow q$(B)

Premise 2: $q \Rightarrow r$(B)

P	Q	r	$p \Rightarrow q$	$q \Rightarrow r$	$p \Rightarrow r$	$(p \Rightarrow q) \wedge (q \Rightarrow r)$	$((p \Rightarrow q) \wedge (q \Rightarrow r)) \Rightarrow (q \Rightarrow r)$
B	B	B	B	B	B	B	B
B	B	S	B	S	S	S	B
B	S	B	S	B	B	S	B
B	S	S	S	B	S	S	B
S	B	B	B	B	B	B	B
S	B	S	B	S	B	S	B
S	S	B	B	B	B	B	B
S	S	S	B	B	B	B	B

Conclusion: $p \Rightarrow r$(B)

It can be seen that by drawing conclusions using the syllogism principle, it is valid

G. QUANTITY STATEMENT

There are two kinds of quantifiers, namely universal quantifiers and existential quantifiers.

Universal quantifiers are statements that apply in every or to all.

The quantifier symbol is $\forall(x)$. p(x) reads "for all x applies p(x).

Existential quantifiers are statements that apply to at least one existential quantifier symbolized by $\exists(x)$. p(x) is read as x, is p(x)

Example:

a. $\forall(x)$, $x \in R$, $\sin^2 x + \cos^2 x = 1$

b. $\exists(x)$, $x \in R$, $x + 4 = 7$

Answer:

a. True, because for all x real numbers $\sin^2 x + \cos^2 x = 1$.

b. True, because there is x = 3 , so 3 + 4 = 7

Practice Questions

1. The truth value of $\sim p \wedge q$ is …
2. The negation of the statement "x is more than y" is...

Excerpt from the sentence. "if it rains heavily then Budi gets a cold", is

3. The contraposition of the sentence "if the price of goods rises then people complain" is ....
4. Inverse of implication: “if $a < b$ then $a^2 - b^2 < 0$ is…
5. The converse of the implication: “when the sun rises, the rooster crows” is…

order of truth values of: $(p \Leftrightarrow \sim q) \Rightarrow \sim r$ is…

6. Is known:

Premise 1: $(p \wedge \sim q) \Rightarrow r$

Premise 2: $\sim r$

A valid conclusion from the two premises above is…

7. The negation of: $(\exists x \in R).(x^2 - 2x - 15 < 0) \Rightarrow (-5 < x < 3)$ is…
8. the truth value of the statement: if he is guilty (violates the law) then he is punished is ...

CHAPTER IV
RELATIONS AND FUNCTIONS

This chapter discusses the concept of relations which consists of: definition of relations, representation of relations, inverse relations, operations on relations and composition of relations. It also discusses the function concept which consists of: function definition, origin and result area, odd and even functions, operations on functions and properties on functions.

A. Relation

1. Definition of relation

Let A and B be a non-empty set. To express the relationship between the elements that exist in the sets A and B, an ordered pair set is used. The set of ordered pairs in question is a Cartesian product between sets A and B.

The Cartesian product of sets A and B is the set whose elements are all possible ordered pairs, where the first component of the set A and the second component of the set B. This can be denoted as:$A \times B = \{(a, b) | a \in A \; dan \; b \in B\}$

Definition 1.1

$Binary\ relation\ R\ between\ \ A\ and\ B\ is\ a\ part\ set\ of\ A \times B.$
binary relation R denoted as $R \subseteq (A \times B)$

Example 1

Let $A = \{2,3,5\}\ and\ B =$
$\{2,6,9,10,15\}\ If\ R\ is\ defined\ as\ relation\ from\ A\ to\ B\ with\ (a, b) \in$
$R\ if\ a\ is\ prime\ factor\ from\ from\ b.\ Determine\ R$:

Answer:

Take $2 \in A, Then$ 2 prime factor of 2
2 prime factor of 6
2 prime factor of 10

Take $3 \in A, Then$ 3prime factor of 6

3 prime factor of 9

3prime factor of 15

Take $5 \in A, Then$ 5 prime factor of 10

5 prime factor of 15

Thus obtained

R =$\{(2,2), (2,6), (2,10)(3,6)(3,9)(3,15)(5,10)(5,15)\}$

The origin and the resultant region of a relation may be the same set, in other words the relation is only defined on one set. Suppose the relation on the set P, then$(x, y) \in R$ where$x, y \in P$

Definition 1.2

The relation on the set A is the relation of $A \times A$

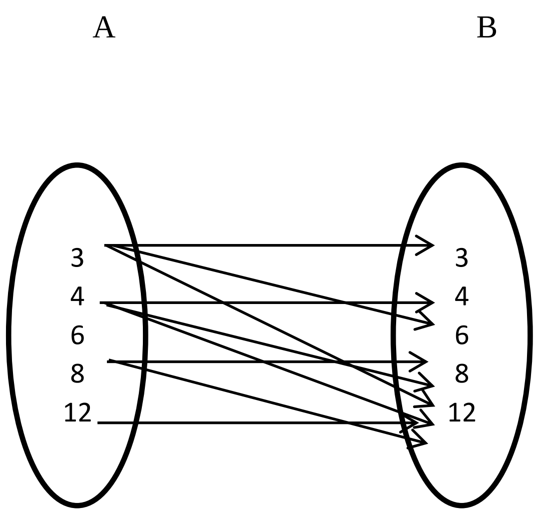
Example 2

Let R be the relation on A defined by R if it divides{3,4,6,8,12}$(x, y) \in xy$

Determine the members of R

Answer

Using an arrow diagram, the relation R can be described as follows:



$$R = \{(3,3)(3,6)(3,12)(4,4)(4,8)(4,12)(6,6)(6,12)(8,8)(12,12)\}$$

2. Relatio representation

Relationships can be represented in various ways, for example by a set of ordered pairs as described previously. In addition, relations can be expressed or represented by tables, matrices, or directed graphs. This time we will discuss the representation of relations in the form of a matrix.

Suppose that R is a relation. The relation R can be represented by a matrix $.A = \{a_1, a_2, \dots\ a_m\}\ dan\ B = \{b_1, b_2, \dots\ b_n\}. M = [m_0]$

$$M = \begin{matrix} & b_1 & b_2 & b_n \\ \begin{matrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{matrix} & \left[\begin{matrix} m_{11} \\ m_{21} \\ \vdots \\ m_{m1} \end{matrix}\right. & \begin{matrix} m_{12} \\ m_{22} \\ \vdots \\ m_{m2} \end{matrix} & \left.\begin{matrix} \dots m_{1n} \\ \dots m_{2n} \\ \vdots\ \vdots \\ \dots m_{mn} \end{matrix}\right] \end{matrix}$$

Which in this case

$$m_0 = \begin{cases} 1 & ,(a_1, b) \in R \\ 0 & ,(a_1, b) \notin R \end{cases}$$

In other words, the matrix element at position (i, j) is 1 if it is associated with and 0 if it is not associated with. The relation representation matrix is an example of a zero-one matrix.$a_i b_j a_i b_j$

3. inverse relation

If there is a relation R from set A to set B, then a new relation can be made from set B to set A by reversing the order of each ordered pair in R.

Definition 1.3

Let R be the relation from set A to set B. The inversion of the relation R, denoted by, is the relation from B to A defined byR^{-1}

$$R^{-1} = \{(b, a)/(a, b)\ \in R\ \}$$

Example 3

Let P = and Q = if we define the relation R from P to Q with (if it is divisible then determine$\{2,3,5\}\{2,6,9,10,15\}p, q) \in R p q R^{-1}$

Answer

The first step is to determine R

R= then, is defined as the inverse of the relation R, i.e. the relation of Q and P with () if is a multiple of$\{(2,2)(2,6)(2,10)(3,6)(3,9)(3,15)(5,10)(5,15)\}R^{-1} q, p \in R^{-1} q p$

So that it is obtained

R^{-1}={(2,2)(6,2)(10,2)(6,3)(9,3)(15,3)(10,5)(15,5)}

4. Operations on relations

In relation, the operations of intersection, union, difference, and equilateral difference apply as applied to sets

Example 4

Let and B =$A = \{p, q, r\}\{p, q, r, s\}$

relation = and relation$R_1\{(p,p)(q,q)(r,r)\}R_2\{(p,p)(p,q)(p,r)(p,s)\}$

is the relation from A to B. Determine, , ,$R_1 \cap R_2$,$R_1 \cup R_2 R_1 - R_2 R_2 - R_1 R_1 \oplus R_2$

Answer

$R_1 \cap R_2$,$= \{(p,p)\}$

$R_1 \cup R_2 = \{(p,p)(q,q)(r,r)(p,q)(p,r)(p,s)\}$

$R_1 - R_2 = \{(q,q)(r,s)\}$

$R_2 - R_1$, =$\{(p,q)(p,r)(p,s)\}$

,$R_1 \oplus R_2$=$\{(q,q)(r,r)(p,q)(p,r)(p,s)\}$

5. Relation composition

Definition 3.5.

Let R be the relation from set A to set B, and S is the relation from set B to set C. The composition of R and S, denoted by S o R is the relation from A to C defined by

$$S\ o\ R = \{(a,c)|a \in A, c \in C, dan\ untuk\ beberapa\ b \in B, (a,b) \in R\ dan\ (b,c) \in S\}$$

6. Relation properties

The properties of the relation are reflexive, symmetrical, and transitive

a. Reflexive

A relation R on a set A is said to be reflexive if$(a,b) \in R\ untuk\ setiap\ a \in A$

b. Symmetry

A relation R on a set A is said to be symmetric if$(a,b) \in R, maka\ (b,a) \in R\ untuk\ semua\ a,b \in A$

A relation R on a set A is said to be unsymmetrical if$(a,b) \in R, dan\ (b,a) \in R\ maka\ a - b, untuk\ semua\ a,b \in A$

c. Transitive

A relation R on a set A is said to be conducting if$(a,b) \in R, dan\ (b,a) \in R, then\ (a,c) \in R\ for\ all\ a,b,c\ \in A$.

Example 5

Suppose P = and R is relations defined on the set P, namely:{2,3,4}

R =$\{(2,2)(2,3)(3,3)(3,4)(4,4)\}$

Is R reflective?

Answer:

Because (2,2) ,(3,3), (4,4) for 2,3,4 then R is reflective$\in R \in P$

Example 6

Suppose A = D and R using the relations defined on the set A, namely:{2,3,4}

R =$\{(2,2)(2,3)(3,2)(3,3)(3,4)(4,3)\}$

Is R symmetric?

Answer

Because (2,3) and (3,2) (3,4) (4,3) then R is symmetric$\in R\ ;\ \in R$

Example 7

Suppose A = and R is relations defined on the set A{1,2,3,4,5}

R =$\{(2,1)(3,1)(3,2)(5,1)(5,2)(5,3)\}$

Is R transitive?

Answer

Pay attention to the table

(a,b)	(b,c)	(a,c)
(3,2)	(2,1)	(3,1)
(5,2)	(2,1)	(5,1)
(5,3)	(3,1)	(5,1)
(5,3)	(3,2)	(5,2)

Because $(a,b)\ dan\ (b,c) \in R$ then, (a,c) $\in R\ for\ a,b,c \in Ra,b,c\ \in A$ then, R is transitive.

7. Equivalent relation

If a relation has reflexive, symmetrical and transitive properties, then the relation is said to be an equivalence relation.

B. Function

1. Function definition

A function$f$is a matching rule that relates each object in a set, called the origin, with a unique value from the second set. The set of values obtained in this way is called the result area (travel) of the function.$xf(x)$

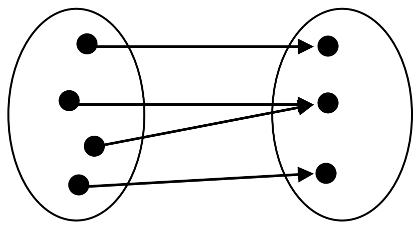


IMAGE 1

This area does not place any restrictions on the sets of origin and result areas. The area of origin may consist of the set of people in your calculus class, the area of values in the form of the set of numbers to be given, and the equivalence rule is the procedure your teacher uses in assigning numbers.(A, B, C, D, F)

To name a function use a single letter like$f$(or or). So , which is read " from " or " on ", indicates the value assigned by to . So, if$gFf(x)fxfxfxf(x) = x^3 - 4$,

$$f(2) = 2^3 - 4 = 4$$
$$f(-1) = (-1)^3 - 4 = -5$$
$$f(a) = a^3 - 4$$
$$f(a+h) = (a+h)^3 - 4 = a^3 + 3a^2h + 3ah^2 - h^3 - 4$$

2. Area of origin and area of production

The equivalence rule is the center of a function, but a function is not completely defined until its domain is given. The domain of origin is the set of elements in which the function gets a value. The result area is the set of values obtained in this way. For example, if is a function with rules and if the origin is specified as (Figure 3), then the result area is the origin area and the rule is to determine the result area. If for a function the domain of the origin is not specified, we always assume that the domain is the largest set of real numbers so that the function rules have meaning and give the value of real numbers. This is called the origin region (natural domain).$FF(x) = x^2 + 1\{-1, 0, 1, 2, 3\}\{1, 2, 5, 10\}$.

Example 8

Given function:, determine the origin of the function (domain)$f(x) = \frac{x-3}{x}\ x \neq 0$

Answer:

The domain of the function is all real numbers except 0 .

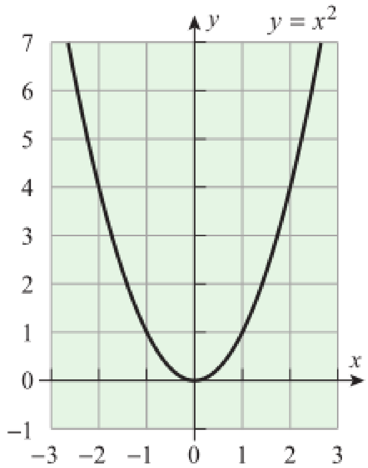
Example 9

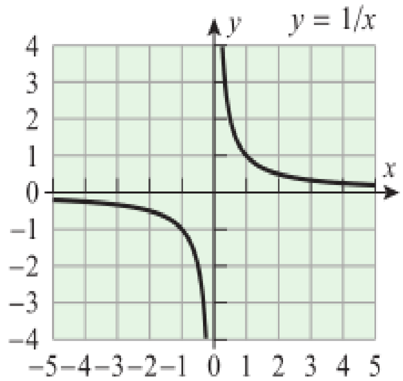
Sketch the graphs of:

a) $f(x) = x^2$

b) $f(x) = {}^{1}\!/_{x}$

Answer:





3. Even and odd function

a. even function

If$f(-x) = f(x)$for all , then the graph is symmetric about the axis . Because the function specifying as the sum of even powers is even.$xyf(x)x$

b. Odd function

If$f(-x) = -f(x)$for all , then the graph is symmetric about the origin. Because a function that gives as the sum of odd powers is odd.$xf(x)x$

Example 10

is$f(x) = x^2$is the function even?

Answer***:***

$f(-x) = (-x)^2$

$= (-x)(-x)$

$= x^2$

$= f(x)$

Then we get , then it is an even function$f(-x) = f(x) f(x) = x^2$

Example 11

is$f(x) = x^3 - x$is an odd function

Answer:

$f(-x) = (-x)^3 - (-x)$

$= -x^3 + x$

$= -(x^3 - x)$

$= -f(x)$

Then we get , then it is an odd function$f(-x) = -f(x) f(x) = x^3 - x$

4. Operation on function

Operations that apply to functions are operations for sum, difference, product, quotient, and exponent.

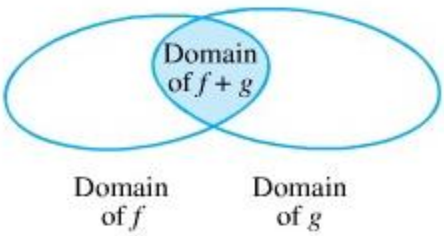
Suppose the function$f$and with the formula$g$

$$f(x) = \frac{x-3}{2}, \quad g(x) = \sqrt{x}$$

We can prove a new function$f + g$by assigning a value to ; that is,$(x) + g(x) = (x-3)/2 + \sqrt{x}$

$$(f+g)(x) = f(x) + g(x) = \frac{x-3}{2} + \sqrt{x}$$

Of course we have to be a little careful about the origin. Obviously it must be a number where and applies. In other words, the origin is the intersection of the origin and$x f g f + g f g$



In addition to functions, we also get new functions and are introduced in an analog way. Assuming that and has a natural origin, we get the following:$f + g f - g,\ f.g,\ f/g f g$

Formula	Place of Origin
$(f+g)(x) = f(x) + g(x) = \frac{x-3}{2} + \sqrt{x}$	[0,)∞
$(f-g)(x) = f(x) - g(x) = \frac{x-3}{2} - \sqrt{x}$	[0,)∞
$(f.g)(x) = f(x).g(x) = \frac{x-3}{2} + \sqrt{x}$	[0,)∞
$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x-3}{2\sqrt{x}}$	(0,) ∞

5. Function properties

Based on the phenomena that occur in the codomain/image, the function consists of three parts, namely a one-to-one function (injective), a one-to-one function (surjective), and one-to-one correspondence (bijective).

a. One-on-one function

Function$f$is said to be one-to-one (into) or injective if no two elements of the set have the same image. In other words, if and are members of the set, then if . If then$A\ abAf(a) \neq f(b)a \neq bf(a) = f(b)a = b$

b. Function$f$is said to be on (onto) or surjective if each element of the set is an image of one or more elements of the set . In other words, all elements are the range of$BA\ Bf$

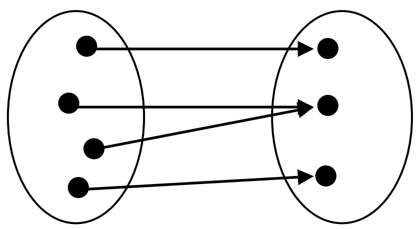


FIGURE 3

c. One-one correspondence

Function f is said to be one-to-one or bijective if the one-to-one function and the function in the origin have exactly one image in the result region. f

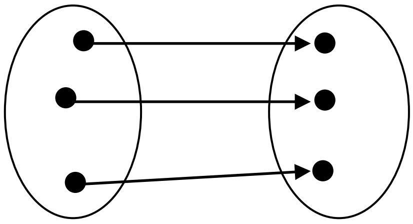


FIGURE 4

PRACTICE

Do the following practice questions correctly

1. Write down the n elements of the relation in $R\{1, 2, 3, 4\}$ that are defined by $(x, y) \in R$ if $x^2 \geq y$.

2. For each relation of $\{1, 2, 3, 4\}$ following, determine whether it is reflexive, symmetrical, asymmetrical, and conducting.

a) $\{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3,4)\}$

b) $\{(2, 4), (4, 2)\}$

c) $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$

d) $\{(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)\}$

3. If R is the relation of $\{(1, 2), (1, 3), (2, 3), (2, 4), (3, 1)\}$ and S is a relation of $\{(2, 1), (3, 1), (3, 2), (4, 2), (4, 2)\}$. Determine $S\ o\ R$ dan $R\ o\ S$.

4. If R= $\{(1, 2), (2, 3), (3, 4)\}$ and S=$(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 4)$ $are\ relation\ from$ $\{1, 2, 3\}$ to $\{1, 2, 3, 4\}$. Define :

a) $R \cup S$

b) $R \cap S$

c) $R - S$

d) $S - R$

e) $R \oplus S$

5. Let R be a relation on the set of ordered pairs of positive integers such that $R\big((a, b), (c, d)\big) \in R$ if and only if $ad = bc$. Show that R is an equal relation.

6. Find the area of natural origin in each case.

a. $f(x)= (4-x^2)/(x2-x-6)$

b. $G(y)= ((y+1)^{(-1)})$

c. □$(\mu)=|2\mu+3|$

d. $F(t) = t^{(2/3)}-$

7. Let $f(x)=(ax+b)/(cx-a)$. Prove that $f(f(x))= x$ provided that $x^2+bc\neq 0$ and $x\neq a/c$

CHAPTER V
ALGEBRA

Algebra developed by Al-Khawarizmi was an expert in mathematics, astronomy, geography, and astrology of Persian origin. He is known as the inventor of algebra and the number zero (0). The real name of Al Khawarizmi is Muhammad Ibn Musa Al Khawarizmi. Al Khwarizmi was born and flourished during the reign of the Abbasid Caliphate while centered in Baghdad (762-1258 AD). At that time, the development of Islam in science was very rapid. During the time of Caliph Harun Al-Rasyid (786-809 AD), Baghdad was known as the center of civilization as well as the center of science. He was born in Bukhara, living in Khawarizm (now Khiva), Usbekistan in 194 H / 780 AD and meninngal in 266 H / 850 AD in Baghdad. Al Khawarizmi as an algebraic teacher in Europe. Al-Khawarizmi is also known by the name of Abu Abdullah Muhammad ibn Ahmad ibn Yusoff, in the Western world Al-Khawarizmi is known as al-Khawarizmi, al-Cowarizmi, al-Ahawizmi, al-Karismi, al-Goritmi, al-Gorismi, and several other spellings.

He once introduced Indian figures and ways of calculating Indian numbers in the Islamic world. Al Khawrizmi was the mathematician who introduced algebra and hisab for the first time. He also studied and produced many popular mathematical concepts that are still valid today.

Algebra (Algebra) is the core of mathematics. In the 21st century, al Khawarizmi's work was translated by Gerhard of Germano and Robert of Chaster in European languages. In 820 AD, the work entitled " Hisab al jibra wa al muqabalah" that appeared earlier and the term algebra has not been introduced. The book **Al-Jabar** (*Al-Kitāb al-mukhtaṣar fī ḥisāb al-jabr wa-l-muqābala*) is a first-published book containing the cystimatic settlement of linear and quadratic notation. This work earned Al Khawarizmi the nickname of the Father of Algebra. This book carries contributions in the language and the word Algebra is taken from the word al-jabr contained in his book. Al-jabr's book was a very large revolution in mathematics. Al Khawarizmi successfully carried out the integration of geometric concepts from ancient Greek mathematics into new mathematical concepts. The results of Al Khawarizmi's thought produced a simultaneous theory that made rational, irrational and geometric magnitudes used as algebraic objects.

Al Khawarizmi is estimated to have died in 850 AD. His work was not only in the field of mathematics, many of his thoughts were influential in other fields of science. One example in the field of geography, he perfected ptolemy's map under the title *Kitāb ṣūrat al-Arḍ* and according to Paul Gallez, this is very useful for determining our position in poor conditions.

Al Khawarizmi also exerted a lot of influence on the development of world science, including:

1. Discovered the concept of algebra that we know today through the book Al-Jabr which contains linear and quadratic equations.
2. The first person explained and re-popularized the use of the number zero (0) and introduced the decimal notation system and the two-sign of defiance.
3. Introducing a negative sign to a number.
4. Create an astronomical calculation table to measure the distance and depth of the earth. This table also became the basis for research in the field of astronomy.
5. The model of world mapmaking is written in the book ṣūrat al-Arḍ which western geographers used in drawing maps.
6. Discovered the concept of a timing device with the shadow of sunlight in a book of sundials.
7. Finding the basic concepts of algorithms through the discussion of the rules of performing arithmetic using Hindu-Arabic numbers and systematic solutions.

In this chapter, algebraic material will be discussed related to the linear equations of one variabell and the linear inequality of one variable. Before studying the linear equations of one variabel and the linear inequality of one variabel, we must understand the meaning of kalimat statements, closed sentences, and open sentences.

1. Statement Sentence

In mathematics there are several types of s-types of sentences that have been knowni, such as: news sentences, command sentences, and question sentences. There are several examples of such sentences. Have you ever answered your question or your teacher?' If ever, what is the answer you put forward ? True or false?

If you answer completely, it's best to answer a sentence.

For example : "How many students are in your class?"

An example of the answer is " Many students in my class there are 40 people.

Consider the following sentence :

a. There are 3 sepak takraw players in a team

b. The country's currency of the United States is the Dollar

c. Cubes are space constructs

d. 23 is a prime number

e. -5 > -10

f. $\frac{2}{5}+\frac{4}{8}=\frac{23}{40}$

g. Odd number multiplied by even number the result is an odd number

From some of the examples above, a sentence that is a true sentence or a wrong sentence? A sentence that can be determined to be true or false is called a **statement sentence**.

True sentences or false sentences are called *closed statements or sentences.*

a. A *wrong* sentence is a sentence that states things that do not correspond to generally accepted reality/circumstances.

b. *The correct* sentence is a sentence that expresses things that correspond to the circumstances, the generally accepted reality.

c. Sentences that are of true or false value are called *closed sentences* or are often called *statements.*

2. Open Sentence

Consider the following sentence stated in a matter:

One day Ricki was carrying a bag containing a book. Before the bag was opened Ricki told his friend "there are 9 books in the bag". What do you think of Ricki's remarks? true or false ?

Pay attention to the sentence " *many books in the bag there are 9 pieces*"

Are you able to determine the sentence is true or false?

The sentence Ricki said can be determined that the sentence is true or false. The sentence that Ricki said could not be determined the truthful value so that we checked the bag owned by Ricki. After we check Ricki's bag to make sure the number of books in the bag, there are two possibilities, namely there are 9 books in Ricki's bag that we can value sentences of correct value and the tone of the book that does not amount to 9 pieces that we can value wrong sentences.

Example: "10 *plus a number is* 16"

The sentence in the example, we cannot determine the sentence is true or false, because the word "a number" in the sentence is not yet known. The sentence will be of true or false value depending on what value is given *to "a number*". If we replace the value of "*a number*" with the number 6 then the sentence will be of correct value, because 10 plus 6 equals 16. Conversely if we replace the value of "*a number*" with 3 then the sentence will be of false value, because 10 plus 3 equals 13. Sentences that cannot be determined to be true or false are called open sentences. In mathematics, the sentence "*a number*" that has not yet been determined is called a variable or changer. A variable or changer is symbolized by a lowercase letter , , , , or any other form. If "$xyana$ *number*" is replaced with, the sentence "10x *plus a number the result is* 16" can be written to be $10 + x = 16$.

Note the following sentence :

a. $x - 6 = 15$

b. $3 + x = 10$

Can't say the sentence is true or false yet, because the value is not yet known. When the symbol is replaced with the symbol of the number, then it can be said that the sentence is true or false. If replaced with "15" , the sentence is worth being false ; but when replaced with 21 , the sentence is of correct value. The coat of arms can also be replaced using lowercase letters in other alphabets, namely ; $(x)(x)(x)(x)(x)a$, , , , ... ,,$bcxyz$ from the above form

$x - 6 = 15$ (open sentence)

$15 - 6 = 15$ (statement sentence is worth being false)

$21 - 6 = 15$ (statement sentences are of true value)

$3 + 7 = 10$ (statement sentences are of true value)

Letters are on and called $xx - 6 = 153 + x = 10$*variables* (changers), while 6, 15, 3, and 10 are called constants.

a. An open sentence is a sentence that contains variables and cannot yet be known for its truth value.

b. A variable (changer) is a symbol (symbol) in an open sentence that can be replaced by any member of a set that has been defined

c. A constant is a symbol that represents a number

3. Open Sentence Completion

Each open sentence contains variables that can be replaced with one or more predefined members. The substitution of a variable that makes an open sentence into a true-valued sentence is called a settlement.

Example:

a. $x + 12 = 22,$

The correct *replacement x* is 10.

So, the completion of the open sentence is $x = 10$

b. Known is an odd number and is a coefficient on the number , , , . Determine the value that meets !$xx1a2b3c4dx$

The correct substitutes are 1 and 3$x,$

So, the completion is 1 and 3.

A. One Variable Linear Equation

1. Definition of Similarity

Similarity is a statement sentence that contains the same relationship as (=). That is, the sentence is already clearly the value of the truth whether true or false. Example:

a. $5 + 6 = 9$. (falsely valuable similarities)

b. $7 - 4 = 3$. (true value similarities)

However, not all similarities have no variables, or in other words, not all open sentences containing the same relationship as (=) are similar. Consider some of the following examples.

a. $x - 4 = x - 4$

b. $3x + 6 = x + x + x + 6$

In the example above that is, $x - 4 = x - 4$ and $3x + 6 = x + x + x + 6$ is a similarity, because if *x* is replaced with any number, then always the sentence is obtained correctly. Thus and $x - 4 = x - 4$ $3x + 6 = x + x + x + 6$ not an open sentence, because it is a true sentence or called **similarity**.

2. Pengertian Linear Equation One Variable

Pay attention to the open sentence $a - 3 = 7$.

The open sentence is connected by an equal sign (=). Next, an open sentence connected by an equal sign (=) is called *an equation.*

An equation with one variable of rank one is called *a one-variable linear equation*.

If the equation is replaced with then the equation is of correct value. As for if a is replaced by a number other than 10 then the equation a–3=7 is of false value. In this case, the value is called the solving of a linear equation . Furthermore, the set of solving of linear equations is $aa - 3 = 7a = 10a = 10a - 3 = 7a - 3 = 7\{10\}$,

Problem 1 :

Sherly bought 20 pencils.

a. When he got home, his sister asked for a few pencils, it turned out that the pencils were left with 17 pieces left, how many pencils did her sister ask for?

b. If Sherly needs 8 pencils, and the rest is evenly distributed to her four younger siblings. How many pencils did each of his younger siblings receive ?

In the above problem:

a. If many of the pencils requested by Sherly's sister are fruity, then a sentence is obtained : $x20 - x = 17$

Which is the variable or change in the sentence ?

- How many variables are there?
- Is it an open sentence? $20 - x = 17$
- In sentences using hyphens " = "$20 - x = 17$
- In the highest rank sentence of the variable is one.$20 - x = 17$

Open sentences that use hyphens " = " are called **equations**. If the highest power of the variable of an equation is **one** then the equation is called a **linear equation.** A linear equation containing only one variable is called a **one-variable linear equation (PLSV).** So is one example of PLSV $20 - x = 17$

b. If the many pencils obtained by each of Sherly's younger siblings are n, then the equation is obtained $8 + 4n = 20$

- If n is replaced with 5, then the sentence becomes : and is of false value $8 + 4(5) = 20$
- If n is replaced with 3, then the sentence becomes : and is of true value $8 + 4(3) = 20$

The substitute for it to be true is 3. $n8 + 4n = 20$

The substitute for a variable (changer) so that the intersection becomes correct is called The solution of the equation, while the set containing all the solutions is called the set of solutions.

1. **Solving One-Variable Linear Equations**

Suppose, Deny wants to answer by example the problem of a linear equation of one variable 3x = 9 with x members of the natural number. He replaced with 3 so that the open sentence became correct. $x3x = 9$

$3x = 9 \Rightarrow 3$. , is the completion/answer of PLSV $33 = 9x = 3x$=9. So, the set of completions of is $3x = 9\{3\}$.

The solving of a one-variable linear equation is a substitute number of variables in the area of the equation definition that makes the equation a true statement.

a. Solving Equations by Substitution

Solving equations by substitution means solving equations by **replacing *variables*** with predetermined numbers, so that the equation becomes a **true sentence**.

Example:

Determine the solution of the equation $2x - 1 = 5$

Answer:

For then (is the wrong sentence) $x = 1.2 \times 1 - 1 = 5$.

For then (is the wrong sentence) $x = 2.2 \times 2 - 1 = 5$,

For then (is the $x = 3.2 \times 3 - 1 = 5$ **correct** sentence).

For then (is the wrong sentence) $x = 4.2 \times 4 - 1 = 5$,

So, the settlement is $x = 3$

b. Solving Equations by Adding or Subtracting Both Fields by Equal Numbers

Note the following similarities!

a. $3 + 4 = 7$ (**true** sentence)

$3 + 4 + 10 = 7 + 10$ (both segments plus 10)

$17 = 17$ (**true** sentence)

b. $5 + 6 = 11$ (**true** sentence)

$5 + 6 - 3 = 11 - 3$ (both segments minus 3)

$8 = 8$ (**true** sentence)

It turns out that the similarity is still of correct value if both fields *are added or subtracted by the same number*.

Next consider the following equations !

a. $x + 6 = 10$

$x + 6 - 6 = 10 - 6$ (both segments minus 6)

$x - 0 \quad = 4$

$x \quad = 4$

Checking $x + 6 = 10$

For , then ($x = 44 + 6 = 10$**correct** sentence).

So the solution is $x = 4$,

b. $x - 7 = -12$

$x - 7 + 7 = -12 + 7$ (both segments plus 7)

$x - 0 \quad = -5$

$x \quad = -5$

Checking $x - 7 = -12$

For , then ($x = -5 - 5 - 7 = -12$**correct** sentence).

So the solution is $x = -5$,

c. Solving Equations by Multiplying or Dividing Both Segments of the Equation by Equal Numbers

Note the following similarities!

1) $3 \times 7 = 21$ (true sentence)

$3 \times 7 \times 2 = 21 \times 2$ (both fields multiplied by 2)

$42 = 42$ (**true** sentence)

2) $2x \times 5 = 20$

$\frac{1}{5} \times 2x \times 5 = \frac{1}{5} \times 20$ (Both fields are multiplied $\frac{1}{5}$)

$2x = 4$

$\frac{1}{2} \times 2x = \frac{1}{2} \times 4$ (Both fields are multiplied $\frac{1}{2}$)

$x = 2$

Prove:

$2x \times 5 = 20$

For , then $x = 22(2) \times 5 = 20$

$4 \times 5 = 20$

$20 = 20$ (**true** sentence)

So, the solution is $x = 2$,

It turns out that the equality sentence is still of correct value if the two segments are multiplied or divided by the same number.

d. Equation Solving Graph with one Variable

On the number line, the solving graph of an equation is spliced with a noktah or point. Consider the completion of the following equations and their graphs!

$2x - 1 = 5$

$2x - 1 + 1 = 5 + 1$ (both segments plus 1)

$2x = 6$

$\frac{2x}{2} = \frac{6}{2}$ (both segments divided by 2)

$x = 3$ **(true sentence)**

The settlement is $x = 3$.

Prove:

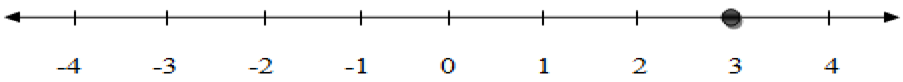
$2x - 1 = 5$

For, then $x = 32(3) - 1 = 5$

$5 = 5$ **(true sentence)**

So, the solution is $x = 3$,

The solving graph of the above equation is:



e. Solving fractional shape equations

A fractional form equation is a compound whose variable contains a fraction, or whose constant number is in the form of a fraction or both contain a fraction.

To solve fractional equations in an easier way, first convert the equation into another equation that is equivalent but no longer contains fractions. This can be done by multiplying the two segments of the equation by the Smallest Common Multiple (**KPK**) of the denominators.

In addition, fractional form equations can also be completed without changing the shape of the equation.

Example:

Determine the solution of the equation $\frac{2}{5}(3x - 2) = 6$.

Answer:

$$\frac{2}{5}(3x - 2) = 6$$

$5 \times \frac{2}{5}(3x - 2) = 5 \times 6$ Both internodes are multiplied by 5

$2(3x - 2) = 30$

$6x - 4 = 30$

$6x - 4 + 4 = 30 + 4$ Both internodes plus 4

$6x = 34$

$\frac{6x}{6} = \frac{34}{6}$ both segments divided by 6

$$x = 5\frac{4}{6}$$

So the solution is $5\frac{4}{6}$

4. Application of Equations in Life

To solve the questions in everyday life in the form of stories, the following steps can help make it easier to solve.

a. If you need a diagram (sketch), for example for one related to geometry, make a diagram (sketch) based on that sentence of the story.

b. Translating story sentences into mathematical sentences in the form of equations.

c. Solving the equation.

Example:

1. Umar and Ali are brothers. Today Ali has his 6th birthday. Currently, Umar's age is 10 years older than Ali's. What is Umar's current age?

Answer:

Umar's age is older than Ali's.

Ali's current age is 6 years old.

It is regrettable that Umar's current age is x years.

So

x = Umar's current age

$x - 10$ = Ali's current age

x = Ali's current age

So that

$$x - 10 = 6$$

$x - 10 + 10 = 6 + 10$ (both segments plus 10)

$$x = 16$$

So, Umar's current lifespan is 16 years.

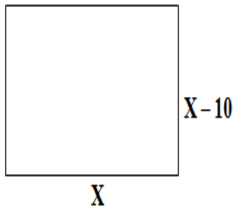
2. Jodi has a fish pond in front of her house in a rectangular shape. The width of the fish pond is 10 cm shorter than its length. If the circumference of the fish pond is 3.8 m, the area of the fish pond is asked.

Answer:

Suppose the length of a fish pond = X

Thus, the width of the fish pond = $X - 10$,

Thus, the image appears:



Mathematical models are and $p = Xl = X - 10$

So that

$K = 2\,(p + l)$

$$380 = 2\,(x + x - 10)$$

Settlement:

$K = 2$⇨$\; + l)$

$380 = 2\,(x + x - 10)$

$380 = 2\,(2x - 10)$

$380 = 4x - 20$

$380 + 20 = 4x - 20 + 20$ (Both segments plus 20)

$400 = 4x$

$x = 100$

So, the length of such a pond is 100 cm².

Broad =⇨$, l$

$= x\,(x - 10)$

$= 100\,(100 - 10)$

$= 100\,,90$

$l = 9000\;cm^2$

So, the area of such a pond is 9000 cm² or 0.9 m².

Example:

Specify the following set of equivalents with the change in the set of integers.

1. $4x + 9 = 3x + 7$

$\leftrightarrow\; 4x + 9 - 9 = 3x + 7 - 9$ (Each segment is reduced by 9)

$\leftrightarrow\; 4x = 3x - 2$

$\leftrightarrow\; 4x - 3x = 3x - 3x - 2$ (Each segment is subtracted $3x$)

$\leftrightarrow\; x = -2$

HP = $\{-2\}$

2. $3c + 9 = 6c - 6$

$\leftrightarrow\; 3c + 9 - 9 = 6c - 6 - 9$ (Each segment is reduced by 9)

$\leftrightarrow\; 3c = 6c - 15$

$\leftrightarrow\ 3c - 6c = 6c - 6c - 15$ (Each segment is subtracted 6c)

$\leftrightarrow\ -3c = -15$

$\leftrightarrow = \frac{-3c}{-3}\frac{-15}{-3}$ (Each segment is divided – 3)

$\leftrightarrow\ c = 5$

HP = {5}

Exercise

Work on the following questions in your assignment book

1. Determine which is a linear equation of one variable and give its reasons.

a) $x + y + z = 20$

b) $3x^2 + 2x - 5 = 0$

c) $x + 9 = 12$

d) $3a - 6 = 7 + a$

e) $2x + y = 1$

f) $3x = 1$

g) $3xy + 2 = 5$

h) $\frac{1}{3}(2y - 8) = 4$

2. Specify the set of completions below.

a) $9 - 3r = 6$

b) $q + 7 = 12$

c) $7a = 3a + 8$

d) $2x + 9 = 3x + 11$

e) $2a - 1 = 3a - 5$

f) $1 = 9 + x$

g) $4 + p = 3$

h) 2 – z = z – 3

B. One-Variable Linear Inequality

In everyday life, of course you have heard sentences like the following.

- Hanifa's weight is more than 50 kg.

- One of the requirements to become a member of the TNI is that the body is not less than 165 cm.
- A Unsri student bus can carry no more than 35 people.

1. Definition of Inequality

An inequality is always characterized by one of the following hyphens.

"<" to state *less than*.

">" to state *more than*.

" ≤ " to state *no more than* or *less than*

or equal to.

" ≥ " to express *no less than* or *more than or equal to*.

Open sentences that use hyphens : <, >, ≤, or ≥ are **inequalities.** An inequality that contains one variable and the power of the variable is one is called **a linear inequality of one variable.**

Example **:**

a. 3 less than 5 written $3 < 5$

b. 8 more than 4 written $8 > 4$

c. x no more than 9 written $x \leq 9$

d. twice y not less than 16 written $2y \geq 16$

2. Definition of One-Variable Linear Inequality

For example *a*, *b* is a real number, with $a \neq 0$,

A One-Variable Linear Inequality (PtLSV) is an open sentence that has a variable expressed by the form:

$ax + b > 0$ or or or .$ax + b < 0ax + b \leq 0ax + b \geq 0$

Issue 1

In her daily life, Beni finds sentences like the following:

Students who participate in remedial are students whose grades are less than 6.

a) Beni's math score is 5. Is Beni remedial? Why? Give your reasons.

b) Beni's math score is 7. Is Beni remedial? Why? Give your reasons.

c) Beni's math score is 6. Is Beni remedial? Why? Give your reasons.

Alternative Solutions

The sentence "Students who take part in remedial are students whose score is less than 6" means that students must follow the remedial if the score is below 6. The word *"below 6"* gives a limit of must be lower than the value 6, the value of 6 and above the value of 6 is not included. The steps of converting the above sentence into a mathematical model we do as follows:

a) Suppose *b* is the student's grade.

b) Turn the word 'less than' into a mathematical symbol that is: <.

c) The mathematical model is .$b < 6$

From the above troubleshooting alternatives we find the following:

a. 4 (four) mathematical models that use <, ≤, >, and ≥ symbols. These four symbols (signs) are signs of inequality. The reading of these symbols is:

< : less than

≤ : less than equal to

> : more than

≥ : more than equal to

b. The formed mathematical model has one variable each.

c. The rank of each of its variables is 1.

If the four mathematical models we find are examples of one-variable linear inequality.

3. The Properties of Inequality

The properties of pertidaksamaan are :

a. If on a pertidaksamaan the two segments are ditambah or subtracted by a number that is sama, mwill be obtained a new pertidaksamaan which is equivalent to the pertidaksamaan semula

b. If in sua or pertidaksamaan multiplied byan number positive , mwill be obtaineda new m aan pertidaksa which is equivalent to the pertidaksamaan semula

c. If on a pertidaksamaan multiplied by a negative number , mwill be obtained a new pertidaksamaan which is equivalent to the pertidaksamaan semula when the direction dari sign of non-existencemaan reversed.

d. If the pertidaksamaannya mengandung fractions, the way menysatur it is mengalikan the two segments with the KPK denominators so that the denominator is lost .

4. **Determining the Set of Linear Settlements One variable**

a. If the two indivisional fields are added or subtracted by a number then the inequality sign remains.

Example : What is the value of x that affects the inequality? $4 + x > 1$

Answer:

$$4 + x > 1$$
$$4 - 4 + x > 1 - 4$$
$$x > -3$$

(both segments minus 4)

So, the sign of indifference remains

b. If the two indigitate fields are multiplied or divided by a positive number then the inequality sign remains.

Example : What is the value of x that affects the inequality? $4 - 2x < 8$

Answer:

$$2x < 8$$
$$\frac{2x}{2} < \frac{8}{2}$$
$$x < 4$$

(both segments sections 2)

So, the sign of indifference remains

c. If the two indifference fields are multiplied or divided by a negative number then the inequality sign must be changed (< to >, ≤ to ≥, and vice versa).

Example : What is the value of x that affects the inequality? $-2x \geq 30$

Answer:

$$-2x \geq 30$$
$$-\frac{2x}{2} \geq \frac{30}{2}$$ *(both segments divided by 2)*
$$-x \geq 15$$ *(The two segments are divided -1)*
$$x \leq -15$$

So, the sign of inequality must be changed

Issue 2

Mr. Fredy owns a box car carrying goods with a carrying capacity of no more than 500 kg. Mr. Fredy's weight was 60 kg and he would transport boxes of goods which each box weighed 20 kg.

a. How many boxes can Mr. Fredy transport at most in one transport?

b. If Mr. Fredy is going to transport 110 boxes, at least how many times will the box be used up?

In order for the above problem to be solved by us, we first turn it into the form of a mathematical model.

The steps to change it are:

Suppose: the large number of boxes of goods transported in a box car $x\ =$.

Changing the word 'no more' into a mathematical symbol i.e.: ≤

So the mathematical model is: $20x + 60 \leq 500$

Weight of one box = 20 kg

Heavy $= 20 \times x\, kg = 20\, x$

Mr. Fredy's weight = 60

Overall weight = $20\, x + 60$

The most boxes that Mr. Fredy can transport in a single haul is the largest value of x on the completion of the inequality . Why? Discuss with your friends.$20x + 60 \leq 500$

We do this as follows.

$$20x + 60 \leq 500$$

$20x + 60 - 60 \leq 500 - 60$ *(both segments are subtracted 60)*

$20x \leq 440$ *(both segments divided by 20)*

$$x \leq 22$$

the largest x that meets the inequality is 22. Then the box that Mr. Fredy can transport in one transport is at most 22 boxes $x \leq 22$,

The least box hauling can occur if Mr. Fredy transports 22 boxes on each haul. Do you agree? Discuss with your friends.

The least number of hauls = = 5 times $\frac{110}{22}$, So that many of the least transportation to transport goods as many as 110 boxes is 5 times the transportation.

Example:

Determine the value that meets the following requirements.

a. $2x - 6 \geq 8x + 5$

b. $\frac{3x-1}{4} < \frac{x}{2} - 1$

Alternative Solution :

a.

$$
\begin{aligned}
2x - 6 &\geq 8x + 5 \\
2x - 6 + 6 &\geq 8x + 5 + 6 \\
2x &\geq 8x + 11 \\
2x - 8x &\geq 8x - 8x + 11 \\
-6x &\geq 11 \\
x &\leq -\frac{11}{6}
\end{aligned}
$$

So, the value of x that satisfies the inequality is $2x - 6 \geq 8x + 5x \leq -\frac{11}{6}$

b.

$$
\begin{aligned}
\frac{3x-1}{4} &< \frac{x}{2} - 1 \\
4\left(\frac{3x-1}{4}\right) &< 4\left(\frac{x}{2} - 1\right) \\
3x - 1 &< 2x - 4 \\
3x - 1 + 1 &< 2x - 4 + 1 \\
3x &< 2x - 3 \\
3x - 2x &< 2x - 2x - 3 \\
x &< -3
\end{aligned}
$$

So, the value of x that satisfies the inequality is $\frac{3x-1}{4} < \frac{x}{2} - 1x \leq -3$

The substitution of a variable from an inequality, so that it becomes a true statement is called the completion of a linear inequality of one variable.

Example :

Check the value that meets the inequality $x4x - 2 > 3x + 5$

$$4x - 2 > 3x + 5$$

$\leftrightarrow\ 4x - 2 + 2 > 3x + 5 + 2$ (Each section plus 2)

$\leftrightarrow\ 4x < 3x + 7$

$\leftrightarrow\ 4x - 3x < 3x + 7 - 3x$ (Each segment is subtracted $3x$)

$\leftrightarrow\ x < 7$

Since the fulfilling value is more than 7, then the set of completions of is $x4x - 2 > 3x + 5\{8.9.10.,,,\}$

Exercise

If the variables in the set are integers. Define the set of completions of the following one variable linear inequality,.

a. $3(x - 4) > 5(x - 1)$

b. $4 \leq x + 5 \leq 8$

c. $2(z - 5) > 2(3 - 6z) + 3$

d. $7k < 5k + 4$

e. $2x + 1 < 4x - 5$

f. $3(p - 7) > 2(p - 5)$

g. $-7 < y + 8 \leq 10$

h. $(y - 5) + 6 < (4 - 3y)$

i. $-7u > 5u - 4$

j. $4s + 2 < 2s - 5$

CHAPTER VI

INVERSE FUNCTION AND COMPOSITION FUNCTION

This chapter discusses the concept of inverse and compositional functions consisting of: algebraic functions, composition functions and inverse functions.

A. Algebraic Functions

Types of algebraic operations are often found in the set of real numbers, such as addition, subtraction, multiplication, division and rank. Algebraic operations on real numbers can be applied to the algebra of functions, that is, if known functions f (x) and g (x), and n rational numbers.

Algebraic operations on functions are set as follows:

1. The sum of the functions f (x) and g (x) is written:

(f + g) (x) = f (x) + g (x)

2. The difference between the functions f (x) and g (x) is written:

(f – g) (x) = f (x) – g (x)

3. The multiplication of the functions f (x) and g (x) is written:

(f x g) (x) = f (x) x g (x)

4. The division of functions f (x) and g (x) is written:

$$(x) = \left(\frac{f}{g}\right) \quad \frac{f(x)}{g(x)}$$

5. The power of the function f (x) by the number n is written:

f^n (x) = $\{f(x)\}^n$

Example:

It is known that the functions f and g are determined by the formula f (x) = 2x – 10 and g (x) =. Specify the values of the following functions: $\sqrt{2x-1}$

(f + g) (x) d. (x) $\left(\frac{f}{g}\right)$

(f – g) (x) e. f^3 (x)

(f x g) (x)

Answer:

a. The sum of functions f (x) and g (x) is

(f + g) (x) = f (x) + g (x) = 2x – 10 + $\sqrt{2x-1}$

b. The difference between the functions f (x) and g (x) is

(f – g) (x) = f (x) – g (x) = 2x – 10 – $\sqrt{2x-1}$

c. The multiplication of the functions f (x) and g (x) is

(f x g) (x) = f (x) x g (x) = (2x – 10) () = 2x - 10 $\sqrt{2x-1}$ $\sqrt{2x-1}$ $\sqrt{2x-1}$

The division of the function f (x) with g (x) is

$\left(\frac{f}{g}\right)$ (x) = = $\frac{f(x)}{g(x)}$ $\frac{2x-10}{\sqrt{2x-1}}$

d. Power of the function f (x)

f^3 (x) = {f (x)}^{3} = $(2x – 10)^3$ = $8x^3 – 160x^2 + 800x – 1000$

B. Composition Function

From the two functions f(x) and g(x) can be formed a new function using composition operations. The composition operation is denoted by o (read : composition or roundabout). The new functions that can be formed by the operation of that composition are :

a. (f o g) (x) read : f composition g(x) or f(g(x))

b. *(g o f) (x) read : g composition f(x) or g(f(x))*

Provided that the functions g : A → B are specified with y = g (x) and f : B → C are specified with y = f (x), then The compositional functions f and g are specified by: h(x) = (f o g) (x) = f (g(x)). Likewise, the opposite is provided that the functions f : A → B are specified with y = f (x) and g : B → C is specified with y = g (x), then The compositional functions g and f are specified with h (x) = (g o f) (x) = g (f (x))

For example, the functions f : R → R and g : R → R are determined by the formula f (x) = 4x – 1 and g (x) = 3x.

Specify: a. (f o g) (x)

b. (g o f) (x)

Answer:

a. (f o g) (x)= f (g (x))

= f (3x)

= 4 (3x) – 1

= 12x – 1

b. (g o f) (x)= g (f (x))

= g (4x – 1)

= 3 (4x – 1)

= 12x – 3

The terms of the composition function (f o g) (x) are as follows:

1. The intersection of the resulting area of the function g and f is not an empty set.

$\mathbf{R_g \cap D_f \neq \phi}$

2. The region of origin of the function (f o g) (x) is the subset of the origin region of the function g.

$\mathbf{D_{(f\ o\ g)} \subseteq D_g}$

3. The result region of the composition function (f o g) (x) is the subset of the result region of the function f.

$\mathbf{R_{(f\ o\ g)} \subseteq R_f}$

Example:

There are functions f : R → R and g : R → R which are determined by the following equation, where:

f (x) = 3x + 1 and g (x) = $\sqrt{x}$

Determine the result of:

a. (f o g) (x)

b. (g o f) (x)

c. Origin region (f o g) (x) and yield region (f o g) (x)

d. Origin region (g o f) (x) and yield region (g o f) (x)

Answer:

The origin region f (x) = 3x + 1 is D_f : {x | x R} and the result area R∈$_f$: {y | y R}. as for the origin region of the function g (x) = is D∈ $\sqrt{x}$ $_g$: {x | x 0, x R} and the result area is R≥∈$_g$: {y | y 0, y R}. then the result of: ≥∈

a. (f o g) (x) = f (g (x)) = f () = 3 + 1 $\sqrt{x}$ $\sqrt{x}$

b. $(g \circ f)(x) = g(f(x)) = g(3x + 1) = \sqrt{3x+1}$

c. Origin region (f o g) (x) = $D_{(f \circ g)}$ = {x | x 0, x R}≥∈

Result area (f o g) (x) = $R_{(f \circ g)}$ = {y | y 1, y R}≥∈

It appears that $D_{(f \circ g)} = D_g$ and $R_{(f \circ g)} \subset R_f$

d. Origin region (g o f) (x) = $D_{(g \circ f)}$ = {x | x -1/3 , x R}≥∈

Result area (g o f) (x) = $R_{(g \circ f)}$ = {y | y o, y R}≥∈

It appears that $D_{(g \circ f)} \subset D_f$ and $R_{(g \circ f)} = R_g$

Completing the composition function can also be in a form where the composition function and one of the other functions are known and we will determine the value of the other functions. Suppose it is known function Eg composition function (f o g) (x) = -3x + 4 and f (x) = x + 1. Define the function g(x).

Answer:

$(f \circ g)(x) = -3x + 4$

$f(g(x)) = -3x + 4$

$(g(x)) + 1 = -3x + 4$

$g(x) = -3x + 4 - 1$

$g(x) = -3x + 3$

So the function $g(x) = -3x + 3$

For another example, there is a composition function (f o g) (x) = 2+3x and a function g (x) = x + 2. Define the function f (x).

Answer:

$(f \circ g)(x) = 2+3x$

$f(g(x)) = 2+3x$

$f(x + 2) = 2+3x$

$f(x + 2) = 2 + 3x$

$f(x) + 2 = 2 + 3x$

$f(x) = 2 + 3x - 2$

$f(x) = 3x$

C. Invers Function

The inverse of the function f expressed in the sequential pasanagn form f: A B with f : {(a,b) | a A and b B} is →∈∈$^{f-1}$: B A so that : →$^{f-1}$: {(b,a) | b B and a A}. When the inverse of a function is a function then the inverse of that function is called the inverse function.∈∈

Suppose the set A: {a, b, c, d} and the set B : {1, 3, 4}. Next the function f : A B is determined by f : {(-2,1), (-1,1), (0,3), (1,4)}. Look for the inverse of the function f.→

Answer : The inverse of the function f is $^{f-1}$ = B A is determined by : →$^{f-1}$: {(1,-2), (1,-1), (3,0), (4,1)}.

Some ways to determine the formula of the inverse function $^{f-1}$(x) if the function f (x) is known are:

1. Change the equation y = f(x) in the form f as a function y.
2. The form x as a function y in step 1 and is named $^{f-1}$(y).
3. Change y at $^{f-1}$(y) with x to obtain the form of the inverse function $^{f-1}$(x). Then $^{f-1}$(x) is the formula of the inverse function of the function f (x).

Suppose the following function is a mapping from R to R. define its inverse formula

a. f (x) = 3x + 4

b. f (x) = 3x – 2

Answer:

1. f (x) = 3x + 4

y = f (x) = 2x + 2

x = $\frac{y-2}{2}$

x = $^{f-1}$(y) = $\frac{y-2}{2}$

$^{f-1(}$x) = $\frac{x-2}{2}$

2. f (x) = 3x – 6

y = f(x) = 3x – 6

x = $\frac{y+6}{3}$

x = f-1(y) = $\frac{y+6}{3}$

f-1(x) = $\frac{x+6}{3}$

EXERCISE

1. The functions f and g are determined by the formula

f (x) = x + 1 and g (x) = $\frac{1}{2x-1}$

Specify:

a. (f + g) (x) and (f + g) (3)

b. (f – g) (x) and (f – g) (2)

c. (f x g) (x) and (f x g) (2)

d. $\left(\frac{f}{g}\right)$ (x) and (1)$\left(\frac{f}{g}\right)$

e. f^2 (x) and f^2 (2)

2. The functions f : R → R and g : R → R are determined by the formula :

3. f (x) = x^2 + 3 and g (x) = $\frac{2}{x+2}$

a. Determine the region of origin of the functions f and g

b. Specify (f o g) (x) and (g o f) (x)

c. Specify the region of origin and result of the function (f o g) (x)

d. Specify the region of origin and result of the function (g o f) (x)

4. Define the formula of the inverse function $^{f-1}$(x), the following function:

f (x) = 2x – 1

f (x) = - 1/2 x + 4

f (x) = $^1/_3$ (x – 3)

CHAPTER VII

SQUARE EQUATION AND INEQUALITIES

This sub-chapter discuss the meaning of quadratic equations, finding the root types of quadratic equation, discriminant quadratic equations, the meaning of quadratic inequalities, solving quadratic inequalities.

Understanding Quadratic Equations

A quadratic equation is an open mathematical statement that expresses an equal-to-equal relationship (=) and contains a variable with the highest positive power, namely 2. The quadratic equation has a general form, namely: $ax^2 + bx + c = 0$, provided that a, b, and c are real numbers and $a \neq 0$. The description of the general form of the quadratic equation, x is a variable or variable, a is the coefficient of x^2, b is the coefficient of x, and c is a constant.

Example:

1. **$3x^2 + 13x - 10 = 0$**, based on the general form of the quadratic equation the example meets the requirements. This means that the equation has, $a = 3 \neq 0$ is the coefficient of x^2, $b = 13$ is the coefficient of x, and $c = -10$ is a constant. Then 3, 13, and -10 are members of real numbers.
2. **$x^2 - 4 = 0$.** value $a = 1$, $b = 0$, $c = -4$, If $b = 0$ then the form of the equation becomes $ax^2 + c = 0$. A quadratic equation of this form is called a **perfect quadratic equation.**
3. **$2x^2 + 4x = 0$**. value $a = 2$, $b = 4$, $c = 0$, If $c = 0$ then the form of the equation becomes $ax^2 + bx = 0$. A quadratic equation of this form is called an **incomplete quadratic equation.**

Finding the Roots types of a Quadratic Equation

The way to **solve a quadratic equation** is determine a solution or substitute for a variable in the form of a value, so that the equation is true. The values that satisfy the quadratic equation are the solutions to the quadratic equation known as **the roots of quadratic equation.**

There are several ways to find the roots of a quadratic equation, namely:

1. factoring
2. Complete a perfect square (converted to a perfect square)
3. Using the square root formula

A. Finding the roots of quadratic equation by factoring

To find the roots of a quadratic equation, you can use the factoring method. Factoring or factorization is to express the addition of algebraic terms into the form of multiplication of factors. Factoring quadratic equation by making it into product of two linear equations. This method takes advantage of one of the properties that apply to real numbers. These properties can be stated as follows:

If a, b ∈ R (a and b is a member of the set of real numbers) and then a×b = 0. Then the value a = 0 or value of b = 0. With the following conditions:

1. If the meaning a = 0 and value of b ≠ 0
2. If the meaning b = 0and value of a ≠ 0
3. If the meaning a = 0and value of b = 0

Example of test and discussion:

Find the roots of the following quadratic equation $x^2 - 3x - 10 = 0$ Answer:

Is known : $x^2 - 3x - 10 = 0$

Finding the roots of the quadratic equation by changing the form of the above equation to: $(x - 5)(x + 2) = 0$ (factoring by using one of the properties of the product of two real numbers) assuming (x - 5) = a and(x + 2) = b

Then returns the value: (x - 5) = 0or (x + 2) = 0, x = 5orx = - 2

then, the solution or the roots are x1 = 5and x2 = - 2. In the form of a solution set, it is written as, HP = {5, - 2}.

B. Finding The Roots of Quadratic Equation by Completing Perfect Square (Converted Into Perfect Square)

In finding the roots of a quadratic equation by completing a perfect square with the following steps:

1. Convert the original quadratic equation into the following form: $(x + p)^2 = q$with the condition that $q \geq 0$.
2. Finding the roots of the quadratic equation (x$(x + p) = \pm\sqrt{q}, \quad x = \pm\sqrt{q} - p$

Example of test and discuss:

Find the roots of the following quadratic equation$x^2 - 4x - 2 = 0$

Answer:

The first step:

Converting the original quadratic equation into a perfect quadratic equation as follows $(x + p)^2 = q$ with the following conditions :$q \geq 0$.

$$x^2 - 4x - 2 + \mathbf{4} - \mathbf{4} = 0$$

$$(x^2 - 4x + \mathbf{4}) - 2 - \mathbf{4} = 0$$

$$(x - 2)^2 - 6 = 0$$

$$(x - 2)^2 = 6$$

Second step:

Find the roots of the quadratic equation $(x + p) = \pm\sqrt{q}, x = \pm\sqrt{q} - p$

$$(x - 2) = \pm\sqrt{6}$$

$$(x - 2) = \sqrt{6}\ atau\ (x - 2) = -\sqrt{6}$$

$$x_1 = 2 + \sqrt{6}\ atau\ x_2 = 2 - \sqrt{6}$$

Then , the roots of solving the quadratic equation above are x_1= 2 + 6 and x_2= 2 - 6. Written $HP = \{2 + \sqrt{6}, 2 - \sqrt{6}\}$.

E. Finding The Roots of Quadratic Equation Using Quadratic Root Formula

Let , a, b, and care real numbers and $a \neq 0$. Then, the roots of the quadratic equation $ax^2 + bx + c = 0$are determined by:

E. Finding the Roots of a Quadratic Equation with the Quadratic Root Formula

Let , a, b, and $c$are real numbers and $a \neq 0$. Then, the roots of the quadratic equation $ax^2 + bx + c = 0$are determined by:

$ax^2 + bx + c = 0 \leftrightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = \frac{0}{a}$, both sides are multiplied$\frac{1}{a}$

$$\leftrightarrow x^2 + \frac{b}{a}x = \frac{0}{a} - \frac{c}{a}$$

$\leftrightarrow\ x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2=-\frac{c}{a}+\left(\frac{b}{2a}\right)^2$, both sides are added $\left(\frac{b}{2a}\right)^2$ to turn into a perfect quadratic equation

$$\leftrightarrow\ \left(x+\frac{b}{2a}\right)^2=\left(\frac{b}{2a}\right)^2-\frac{c}{a}=\frac{b^2}{4a^2}-\frac{c}{a}=\frac{b^2-4ac}{4a^2}$$

$$\leftrightarrow\ \left(x+\frac{b}{2a}\right)=\pm\sqrt{\left(\frac{b^2-4ac}{4a^2}\right)}$$

$$\leftrightarrow\ x_{1.2}=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$\boldsymbol{x_{1.2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$

Example of test and discussion:

Find the roots of the following quadratic equation $x^2\ -\ 6x\ +\ 8\ =\ 0$

Answer:

It is known that based on the quadratic equation, the value of $a\ =\ 1$, $b\ =\ -6$, $c\ =\ 8$

$$x_{1.2}=\frac{-(-6)\pm\sqrt{(-6)^2-4.1.8}}{2.1}=\frac{6\pm\sqrt{36-32}}{2}=\frac{6\pm\sqrt{4}}{2}=\frac{6\pm 2}{2}$$

then, the roots are $x_1=\frac{6+2}{2}=\frac{8}{2}=4$ and $x_2=\frac{6-2}{2}=\frac{4}{2}=2$

Written $HP\ =\ \{2,\ 4\}$.

E. Finding the Roots of a Quadratic Equation with the Quadratic Root Formula

Let , a, b, and c are real numbers and $a\ \neq\ 0$. Then, the roots of the quadratic equation $ax^2\ +\ bx\ +\ c\ =\ 0$ are determined by:

$ax^2\ +\ bx\ +\ c\ =\ 0\ \leftrightarrow\ x^2+\frac{b}{a}x+\frac{c}{a}=\frac{0}{a}$, both sides are multiplied $\frac{1}{a}$

$$\leftrightarrow\ x^2+\frac{b}{a}x=\frac{0}{a}-\frac{c}{a}$$

$\leftrightarrow\ x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$, both sides are added $\left(\frac{b}{2a}\right)^2$ to turn into a perfect quadratic equation

$$\leftrightarrow\ \left(x + \frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a} = \frac{b^2}{4a^2} - \frac{c}{a} = \frac{b^2 - 4ac}{4a^2}$$

$$\leftrightarrow\ \left(x + \frac{b}{2a}\right) = \pm\sqrt{\left(\frac{b^2 - 4ac}{4a^2}\right)}$$

$$\leftrightarrow\ x_{1.2} = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\boldsymbol{x_{1.2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Example questions and discussion:

Find the roots of the following quadratic equation$x^2 - 6x + 8 = 0$

Answer:

It is known that based on the quadratic equation, the value of $a = 1, b = -6, c = 8$

$$x_{1.2} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4.1.8}}{2.1} = \frac{6 \pm \sqrt{36 - 32}}{2} = \frac{6 \pm \sqrt{4}}{2} = \frac{6 \pm 2}{2}$$

So, the roots are $x_1 = \frac{6+2}{2} = \frac{8}{2} = 4andx_2 = \frac{6-2}{2} = \frac{4}{2} = 2$

written$HP = \{2,\ 4\}$.

F. Discriminant Quadratic Equation

Discriminant is a value that determines the properties of the roots of a quadratic equation. So the root type of the quadratic equation can be determined by knowing the value of the discriminant or can be denoted by $\boldsymbol{D}$ **.** The general form of the discriminant equation is $\boldsymbol{D = b^2 - 4ac}$**.**

The types of roots of the quadratic equation based on the discriminant value are as follows:

1. If the discriminant value is greater than 0, $D > 0$, then the quadratic equation has two different real roots.

a. If *D* is a perfect square, then **both roots are rational**.

b. If D is not a perfect square, then **both roots are irrational**.

2. If $D = 0$then the root of the quadratic equation has two equal roots (twin roots) real and rational.

3. If $D < 0$then the quadratic equation has no real roots or both roots are not real (imaginary).

Example of test and discussion:

Find the discriminant value of the following quadratic equation$2x^2 - 5x + 2 = 0$

Answer:

Given : $2x^2 - 5x + 2 = 0$, with the following values:$a = 2, b = -5, c = 2$

Then the discriminant value is:

$$D = b^2 - 4ac$$

$$D = (-5)^2 - 4(2)(2)$$

$$D = 25 - 16$$

$$D = 9$$

Since the value of $D > 0$and $D = 9 = 3^2$is a perfect square, the quadratic equation $2x^2 - 5x + 2 = 0$has two distinct and rational real roots.

G. Sum and Product of Roots - Roots of Quadratic Equations

In the previous discussion, we have discussed how to find the roots of a quadratic equation. Meanwhile, in the discussion of this sub-chapter, we will calculate the sum and product of the roots of a quadratic equation. Based on the square root formula, it has the solution roots as follows $\boldsymbol{x_1 = \frac{-b+\sqrt{b^2-4ac}}{2a}}and\boldsymbol{x_2 = \frac{-b-\sqrt{b^2-4ac}}{2a}}$

.

Then the formula for the sum of the roots of the quadratic equation is:

$$\boldsymbol{x_1 + x_2 = \frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a} = \frac{-2b}{2a} = \frac{-b}{a}},$$

Mean while the formula for the product of the roots of a quadratic equation is:

$$\boldsymbol{x_1 \, . \, x_2 = \left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right).\left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right) = \frac{b^2-(b^2-4ac)}{4a^2} = \frac{c}{a}}$$

H. Definition of Quadratic Inequality

quadratic inequality is defined as an open mathematical statement associated with an inequality symbol ($>, <, \leq, \geq$)and contains a variable with a positive and highest exponent, namely 2. The quadratic inequality has a general form, namely: $ax^2 + bx + c > 0$or $ax^2 + bx + c < 0$or $ax^2 + bx + c \leq 0$or $ax^2 + bx + c \geq 0$with the conditions a, b, and $c$are real numbers and $a \neq 0$. The description of the general form of the quadratic inequality, $x$is a variable or variable, $a$is the coefficient of x^2, $b$is the coefficient of x, and $c$is a constant.

I. Solving Inequality Using Number Lines

This chapter discusses how to solve quadratic inequalities using a number line. In general, solving quadratic inequalities $ax^2 + bx + c < 0$; $ax^2 + bx + c \leq 0$; $ax^2 + bx + c > 0$; $ax^2 + bx + c \geq 0$ can be determined using a number line diagram through the following steps:

1. Determine **the generator of zero, namely the roots of the quadratic equation** by changing the sign of the inequality until it becomes "equal to".
2. Draw **a zero generator on the number line**, then determine the sign of each interval by substituting any number in each interval into the equation on the left side. Write $(+)$if the substitution result is positive and write $(-)$if the substitution result is negative. The sign for each interval is always alternating $(+)(-)$ $(+)$or $(-)(+)(-)$, unless the roots are the same (twin).

If the obtained roots are different, just look for a sign at one interval only, the remaining remainders are written differently following the pattern above. Prioritize intervals containing zeros for easier calculation (if zero is not a zero generator)

3. Determine the solution area or shading. For inequalities " $>$ " $atau$ " $\geq$ ",the solution area is in the interval with a positive sign (+). For inequalities " $<$ " $atau$ " $\leq$ ", the area of the solution that is in the interval has a negative sign (-).
4. Write a solution, which is an interval containing the area of the solution. The solution set is at the ends of the interval

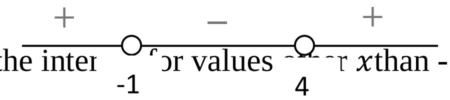
For example, find the solution to the $x^2 - 3x - 4 > 0$following quadratic inequality.

The steps required are as follows:

1. Determine **the generator of zero, namely the roots of the quadratic equation** by changing the sign of the inequality until it becomes "equal to"

$$x^2 - 3x - 4 = 0$$
$$(x + 1)(x - 4) = 0$$
$$x = -1 \; atau \; x = 4$$

2. Draw **the zero generator on the number line** .



We must specify the signs in the interval for values other $x$than -1 and 4, for example:

- $x = -2$then the value of $x^2 - 3x - 4 = (-2)^2 - 3(-2) - 4 = 6$such that the sign in the interval is $x < -1$, xa positive number member, or $x > 0$.
- $x = 1$then the value of $x^2 - 3x - 4 = (1)^2 - 3(1) - 4 = -6$such that the sign in the interval$-1 < x < 4, 1, atau \; x < 0$
- $x = 5$then the value of $x^2 - 3x - 4 = (5)^2 - 3(5) - 4 = 6$such that the sign in the interval $x > 4, x \; bilangan \; positif, atau \; x > 0$.

3. Based on the signs of the interval, what satisfies the inequality $x^2 - 3x - 4 > 0$is $x < -1$or$x > 4$.

4. So, the solution set is$HP = \{ x | \; x < -1 \; atau \; x > 4, \; x \in R\}$.

PRACTICE

1. Find the roots of the quadratic equation $5x^2 + 13x = 6$and calculate the discriminant value of the quadratic equation!

2. The roots of the quadratic equation $x^2 - 2x + 5 = 0$are $\alpha$and . Find the quadratic equation whose roots are (α+2) and +2 ()!

3. The equation $(m - 1)x^2 + 4x + 2m = 0$has real roots. Determine the value $m$that meets!

4. If $p$and $q$are the roots of the quadratic equation $ax^2 + bx + c = 0$, show that $(p - q)^2 = \frac{b^2 - 4ac}{a^2}$!

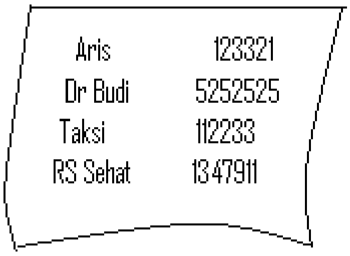
5. Find the solution set for the quadratic inequality $x.(3x + 1) < (x + 1)^2 - 1$

CHAPTER VIII
ARITHMETIC SEQUENCE AND SERIES

This chapter discusses the concept of sequences and series consisting of: number sequences, several number patterns, series, application of sequence and series patterns.

A. Number Sequences

Take a look at the following phone number example!



Regular doctor's checks once a week is another example. If the first control is on the 2nd, then the next controls can be scheduled, namely the 9th, 16th, 23rd, and so on.

The examples of telephone numbers above are easy to remember because each has a certain number pattern. A patterned object will provide certain convenience.

Another example is in the case of bacterial growth. The following is an example of patterned growth. A bacterium reproduces by dividing into 2 every minute. After two minutes it becomes 4, after three minutes it becomes 8, and after four minutes it becomes 16. The number of bacteria growing every minute forms a sequence of 2, 4, 8, 16, 32, . . .By recognizing the pattern of growth of "twice every minute", it can be easily determined how many bacteria each time.

The following sections will present some examples of number patterns and the rules of their formation.

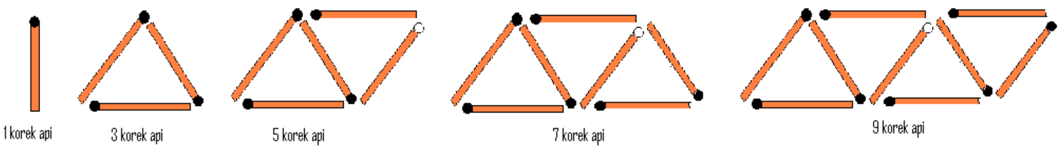
B. Several Number Patterns

Odd Number Pattern

Take a look at the pattern formed by the matchstick arrangement below!

The matchstick

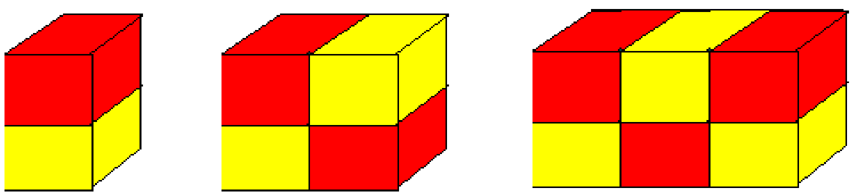
1, 3, 5, 7, 9, . . .



The rule of formation is "plus 2". According to this rule, the next two terms are 11 and 13.

Even Number Pattern

Look at the pattern formed by the lego arrangement below!



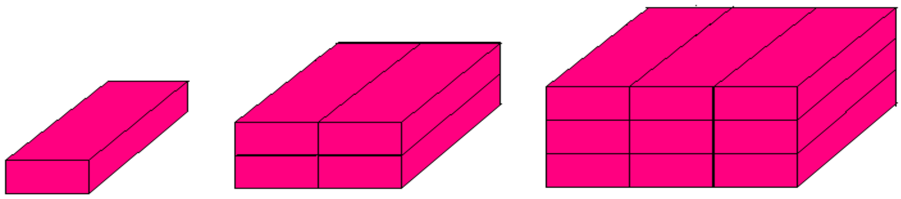
This lego arrangement forms a pattern

2, 4, 6, . . .

The rule of formation is “plus 2”. According to this rule, the next two terms are 8 and 10.

Square Number Pattern

Take a look at the brick layout pattern below!



The brick arrangement above forms a pattern

1, 4, 9, . . .

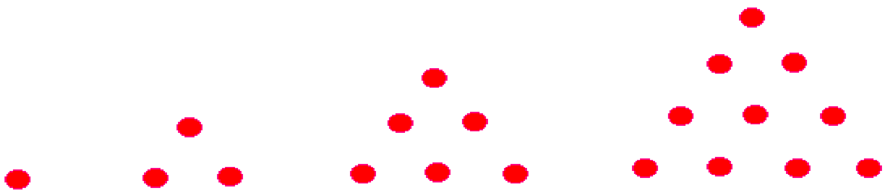
The rule of formation is “square of pattern position”. Therefore, the next two terms are 16 and 25.

Triangle Number Pattern

The sequence of triangle numbers is

1, 3, 6, 9, . . .

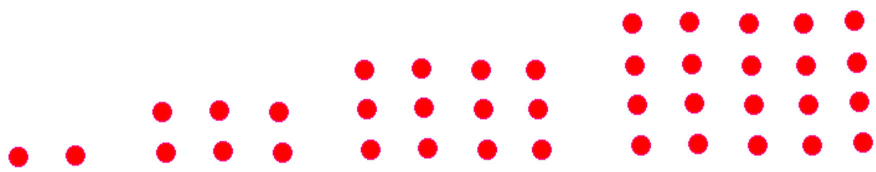
This sequence can be described in the pattern of dots that form the following triangle:



The rule of formation is "add in sequence with 2, 3, 4, and so on". Therefore, by adding 5 and 6 to the previous term, the next two terms are 15 and 21.

Rectangular Number Pattern

Take a look at the following dotted pattern:



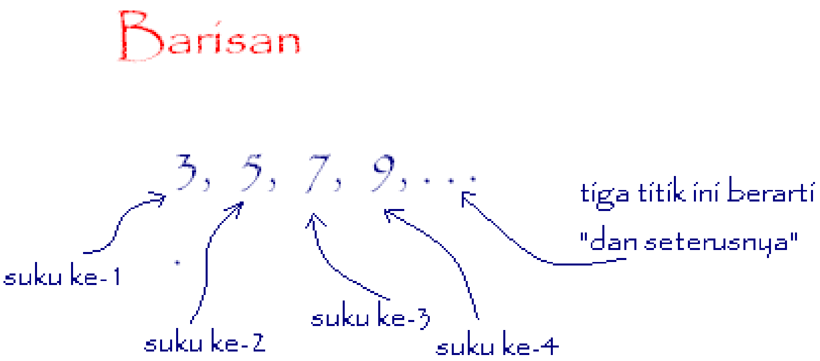
The above pattern represents a rectangular sequence, i.e

2, 6, 12, 20, . . .

Each term of this sequence is "the product of two consecutive original numbers". Therefore, the next two terms are 30 and 42.

The number patterns above are some examples of number patterns that are commonly known. There are of course many other number patterns, including number patterns that we can make ourselves.

From the above patterns, a number sequence can be formed. A **sequence** is a set of numbers arranged in a certain order and formed according to certain rules. Each number that is an element of the sequence is called a **term**. The first number is called the first term, the second number is called the second term, etc. Look at the following illustration!



If the terms of a sequence are continuous, the sequence is called an infinite sequence. On the other hand, if it stops at a certain term, it is called a finite sequence.

Examples

- 0, 1, 2, 3,4, 5, … is an infinite sequence, called a series of non-negative integers
- 1, 2, 3, 4, 5, … is an infinite sequence, called as a sequence of natural numbers.
- 0, 1, 2, 3, 4 is a finite sequence with 5 terms
- 1, 3, 5, 7 is a finite sequence of the first 4 positive odd numbers
- 1, 1, 2, 3, 5, 8, . . . is an infinite sequence that has the pattern "a term is the sum of the previous two terms", such a sequence is called a Fibonachi sequence. Therefore, the next two terms are 13 and 21.

In sequences, order is important and has meaning. Look at the following example!

1, 2, 3, 4, 5, 6, . . .

2, 1, 4, 3, 6, 5, . . .

Even though the constituent numbers are the same but because the order is different, the above two sequences are different.

This order also distinguishes a sequence from a set. In the sequence, the order is considered. While in the set, the order is not considered. The two sequences above are different, but the set { 1, 2, 3, 4, 5, 6, . . .} and {2, 1, 4, 3, 6, 5, . . .} is the same.

To indicate the order in the sequence, we use the index to denote the notation of the terms of the sequence. The n^th^ term of the sequence is denoted by $\mathbf{u_n}$. u1 represents the 1st term, u2 represents the 2nd term, etc. Take a look at the following example!

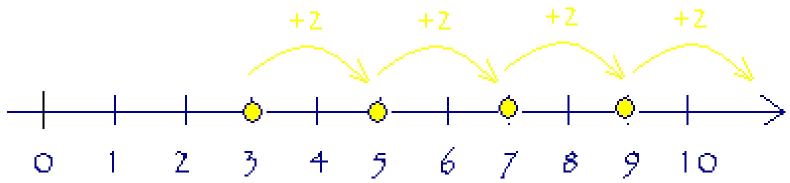
1, 3, 5, 7, …

$u_1 = 1$, $u_2 = 3$, and $u_3 = 5$.

In addition to the order, the important thing about the sequence is the rule or pattern of numbers that compose the sequence. Recognizing the number pattern will make it possible to find the general formula for the n^{th} term. This formula will provide an easy way to determine the value of each term in the sequence.

Example:

Sequence of 3, 5, 7, 9, … starts at 3 and jumps 2 every time.



The rule "starts at 3 and jumps 2 every time" doesn't yet provide a way of getting the 10^{th} term, 100th term, or n^{th} term with any n. Therefore, the n-term formula is needed.

The following will provide an illustration of how to find the formula for the n^{th} term of the sequence in the example above. First, we can see the sequence goes up 2 every time, so we can assume that the formula for the n^{th} term may be "2 x n". Next let's test this rule.

Test Rule: 2n

		ule
		x1 = 2
		x2 = 4
		x3 = 6

The resulting values are close to each other, which are less than 1 each, so we change the formula to 2n + 1.

Test Rule: 2n+1

		ule
		= 2x1 + 1 = 3
		= 2x2 + 1 = 5
		= 2x3 + 1 = 7

The results match, so the formula for the n^{th} term for the sequences 3, 5, 7, 9, … is 2n + 1, or can be determines as $u_n = 2n + 1$.

C. Series

In a certain problem, from a number pattern sometimes it is not enough to just get the formula for the n^{th} term in the sequence of numbers, but further, we want to know the sum of the first n terms. For example, in the case of arranging seats in several rows, where the number of seats in each row follows a certain pattern. So besides wanting to know the number of seats in a certain row, we also want to know the number of seats from the front row to that row. For the sake of this second, it is necessary to discuss the series.

A series is the number of successive terms of a sequence.

If u1, u2, u3, u4, u5, . . .is in a sequence, the sum of the first n terms is denoted by Sn. So that the sum of 1 term, 2 terms, and the first 3 terms are respectively

$S_1 = u_1$

$S_2 = u_1 + u_2$

$S_3 = u_1 + u_2 + u_3$

And in general, the sum of the first terms is

$S_3 = u_1 + u_2 + u_3 + . . . + u_n.$

Example:

In a sequence of square numbers, namely 1, 4, 9, . . . then the sum of the first terms is

$S_1 = 1 = 1$

$S_2 = 1 + 4 = 5$

$S_3 = 1 + 4 + 9 = 14$

$S_4 = 1 + 4 + 9 + 16 = 30$

$S_5 = 1 + 4 + 9 + 16 + 25 = 55.$

From this example, it can also be seen that basically a series of a sequence is also a sequence, which is in the form of

$S_1, S_2, S_3, S_4, S_5, . . .$

which refers to the above example is the sequence 1, 5, 14, 30, 55, . . .

D. Application of Sequence and Series Patterns

Many problems in our daily life are related to sequences and series of numbers. In this case, finding the pattern or formula is the key to being able to solve the problem. Induction method can be used as an alternative to find the pattern or formula.

Example:

In a hall, a number of seats are arranged in several rows. The first row consists of 20 seats. Each subsequent row contains two more seats than the previous row. If there are 20 rows of seats in the hall, then (a) how many people can sit on the seats in the 20th row, and (b) how many people can sit on the seats in the hall.

Solution:

- First, calculate the number of seats for the first, second and third rows, and then use it to formulate the number of seats in the n^{th} row using induction method, as presented in the following table:

	er of seats in the n^{th}	ı
	= 20	0 + 0 = 20 + 2(1 – 1)
	= 22	0 + 2 = 20 + 2(2 – 1)
	= 24	0 + 4 = 20 + 2(3 – 1)
	= 26	0 + 6 = 20 + 2(4 – 1)
		20 + 2(n – 1)

The number of seats in the n^{th} row is 20 + 2(n – 1). So, the number of seats in row 20 is 20 + 2(20 – 1) = 58 seats.

- First, calculate the number of seats for the 1st, 2nd, 3rd, and 4th front rows, and then use it to formulate the number of seats in the n^{th} row by means of induction method, as shown in the following table.

	er of seats in the first n	ı
	= 20	(20 + 0) = 1(19 + 1)
	2 = 42	(20 + 1) = 2(19 + 2)
	4 = 66	(20 + 2) = 3(19 + 3)

	6 = 92	(20 + 3) = 4(19 + 4)
	…… S_n	$4(19 + n)$

From the table above, it can be seen that the number of seats in the n front rows is 4(19 + n).

EXERCISE

Answer the following questions!

1. For each of the following sequences, write the next three terms!

a. 4, 7, 10, 13, ...

b. 1, 8, 27, 64, ...

c. ½, 1, $\frac{3}{2}$, 4, ...

2. Determine the formula for the n^{th} term of the following sequence!

a. 7, 12, 17, 22, ….

b. 0, 3, 8, 15, ...

c. 3, 9, 27, 81, ...

3. Determine the first five terms of the sequence from the following n-term rule! Then make a series!

a. $3n - 2$

b. $n^2 + 1$

c. 4^n

CHAPTER IX
GEOMETRY

This chapter discusses geometric concepts which include: points, rays, angles, and plane shapes (triangles and quadrilaterals).

Basic Geometry: Points, Lines, Angles, and Flat Shapes

In the structure of modern geometry, some terms have been agreed upon and become guidelines for everyone who wants to learn geometry.

Terms in Geometry include:

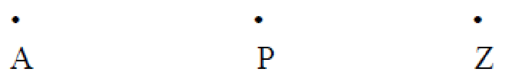
1) Undefined elements (primitive elements)
2) Defined element
3) Axioms and postulates
4) theorem

undefined elements in Geometry, elements that are not defined (*undefined terms of Geometry*) include points (*points*), lines (*lines*), and planes (*planes*). If we (for example) are forced to make a definition for the primitive element, it will be repeated definitions. For example, we (force) create a definition for a point. A dot is something that occupies a place. So, we have to redefine what "something that occupies a place" is. Furthermore, "something that occupies a place" (for example) is defined as "a spot that lies on a plane". Next, we need to define what a "dot" is, and so on. So that in a definition there is another definition and so on so that it will rotate or repeat. Therefore, all concepts that have such properties are included in the category of undefined elements of primitive elements.

Point (*Point*)

In geometry, a point is an abstract concept that is intangible or formless, has no size, has no weight, or has no length, width, or thickness, (*a point has position only*). A dot is an abstract idea or idea that exists only in the mind of the person who thinks about it. To describe or describe a point, a symbol or model is needed. The point representation is a dot (*dot*).

The image or model of a point is usually given a name. The name for a point generally uses a capital letter placed near the point, for example, as in the example below, point A, point P, and point Z.



Painting or drawing a dot can use the tip of an object, for example with the tip of a pencil, pen, compass, or chalk pressed against the writing surface or the surface of the paper or blackboard. If you emphasize the tip of the pencil on the surface of the paper, the black dot that imprints on the surface of the paper is a dot. However, keep in mind that a dot/dot can represent/represent a point but is not a dot. Just like a dot/dot on a map (map) which can represent an area/place/location but is not a locality. A dot is not like a dot, it has a size while a dot has no dimensions.

A-Line

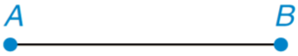
A *line* is an abstract idea or idea whose shape extends in two directions, is not limited or has no endpoint, and has no width or thickness (*a line has length but has no width or thickness*). Drawing a line model can usually be done by making writing utensils on a writing plane, paper, or blackboard with a straight shape. The line model is marked with arrows at both ends which indicates that the line extends in both directions (does not have an endpoint). A line is indicated by a capital letter from its two dots or by a lowercase letter. The line AB is denoted/denoted by $\overleftrightarrow{AB}$).

Line rays (ray)

A line ray is a collection of points which is a combination of certain points on a line and all points on that line that lies on the same side (in non-opposite directions) from that particular point. The symbol for a line of rays whose starting point is A and one of the other points is B is $\overrightarrow{AB}$.

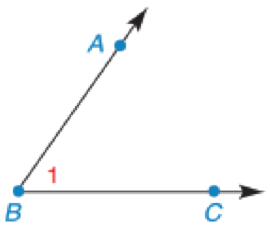
A *line segments*

A line segment is a set of points on a line consisting of points A and B and the points that lie between points A and B. Points A and B are called the endpoints of the line segment. Note that the line segment AB is denoted by $\overline{AB}$.



An *Angle*

An angle (notation) is a combination of two line rays whose starting points are in common

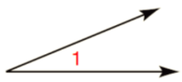


The angle above can be called ∠ABC or ∠CBA or ∠B or ∠1. $\overrightarrow{BA}$ and $\overrightarrow{BC}$ called *side of the angle B*. B called (*vertex of the angle*). The size of the angle is a function of the set of angles to the set of real numbers R. Furthermore, the size of ∠A is denoted by u∠A or m∠A which lies between 0 and 180. In practice, we often use angle units with units of degrees. For example, B u∠B = 60^o

Types of angles based on size

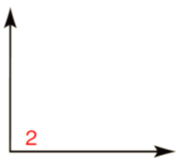
a. An angle that measures less than 90° is called an acute *angle*.

$m\angle 1 = 23°$



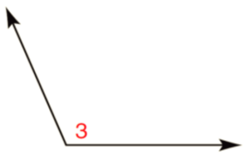
b. If the measure of an angle is exactly 90°, then the angle is a right *angle*.

$m\angle 2 = 90°$



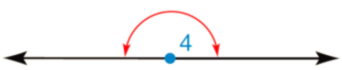
c. If the measure of an angle is between 90° and 180° then the angel is an *obtuse angle*.

$m\angle 3 = 112°$



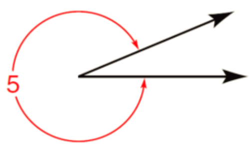
d. An angle that measures exactly 180° is called a straight *angle*. Or a straight angle is an angle whose sides (legs of the angle) are from opposite rays or a straight line (*a straight line*).

$m\angle 4 = 180°$

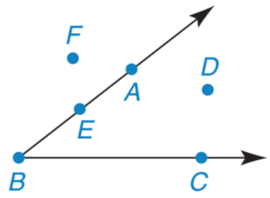


e. The reflex angle is the angle between 180° and 360°

$m\angle 5 = 337°$

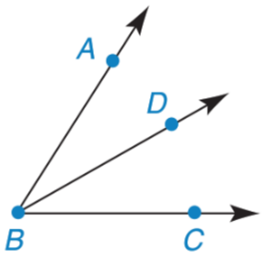


The position of a point concerning an angle



The figure shows "∠ABC" with points D, E, and F. Point D lies in the interior of "∠ABC". Point E lies on "∠ABC". Point F is located on the exterior "∠ABC".

Postulate: If point D lies in the interior of angle ABC (as shown below), then m∠ABD + m∠DBC = m∠ABC



Definition: Two angles are adjacent if they have the same vertex and the common side of the angle that lies between them.

In the picture above "∠ABD " and "∠DBC" are adjacent, while "∠ABC " and "∠ABD" are not sided by side. Give reasons why "∠ABC " and "∠ABD" are not sided by side.

Definition: Two angles are said to be congruent if they have the same measure (measurement)

If "∠A" has the same size as "∠B", then "∠A" is congruent with "∠B" (denoted by "∠A B". So the statement "∠A B" means m" A =" m"∠B"

Definition: Two angles are called complementary if the sum of their measures is 90o. Angles that are pairs of complementary angles are called complements of the other angles.

Example: It is known that m"∠P = " 65o and m"∠Q = " 25o. So "∠P" and "∠Q" we call complementary angles. Then we call "∠P" the complement of "∠Q" and conversely "∠Q" is the complement of "∠P".

Definition: Two angles are said to be complementary if the sum of their measures is 180o. The angle that is a pair of complementary angles is called the complement of the other angles.

Example: It is known that m"∠P = " 105o and m"∠Q = " 75o. Then "∠P" and "∠Q" we call complementary angles. Henceforth "∠P" we refer to as a supplement "∠Q" and conversely "∠Q" is a supplement "∠P".

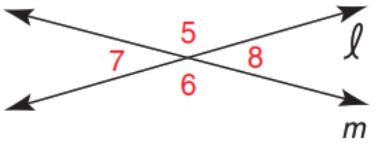
Exercise (Problem 1)

Prove each of the following statements (Theorem):

1. If two complementary angles are at the same angle, then they are congruent
2. If two angles are complementary to the same angle, then they are both congruent
3. . both are congruent
4. If two angles are complementary to two congruent angles, then they are congruent

Definition: Vertical angles (*vertical angles*) are two angles formed by two intersecting lines, with non-adjacent or opposite positions.

Check out the following picture!



From the picture above, "∠5 " is opposite "∠6" and "∠7 " is opposite "∠8 "

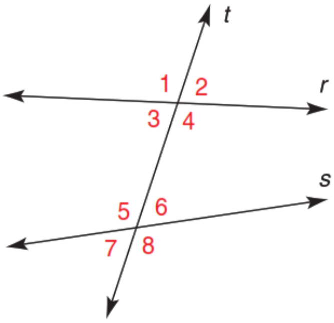
Theorem: "Measures of the angles of opposite angles are congruent"

Proof of the Theorem as an Exercise for the reader (Problem 2)

Hint: Use the rule of complementary angles.

Transverse

Line A transverse line is a line that intersects two (or more) other lines at different points. These lines lie in the same plane.



In the figure above, the line t is the transverse line for lines r and s. The angles formed between r and s are called interior angles →"∠3", "∠4", "∠5", and "∠6". Whereas outside r and s are called exterior angles →"∠1", "∠2", "∠7", "∠8"

From the figure above, the pairs of corresponding angles are "∠1" and "∠5"; "∠2" and "∠6"; "∠3" and "∠7"; "∠4" and "∠8"

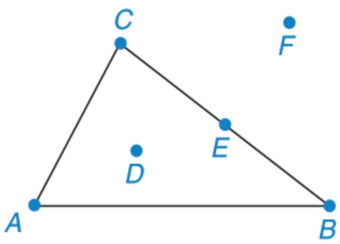
Pairs of alternate interior angles are "∠3" and "∠6"; "∠4" and "∠5"

Pairs of alternate exterior angles are "∠1" and "∠8"; "∠2" and "∠7"

Two-dimensional figure

Triangle

A triangle (symbolized by) is the union of three line segments defined by three non-collinear (non-linear) points.



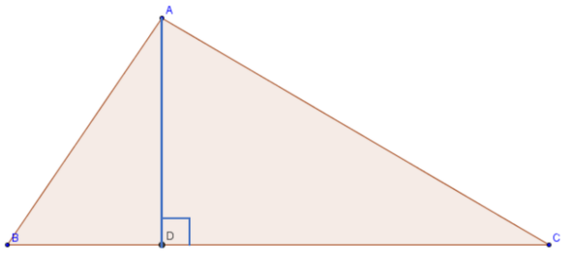
In the picture above, each point A, B, and C is called the vertex $\triangle ABC$ (vertex of triangle). The segments ($\overline{AB}$) , ($\overline{BC}$) , and ($\overline{AC}$) are called the sides of "ABC" (side of triangle). Point D is on the interior of $\triangle ABC$, point E is on $\triangle ABC$, and point F is on the exterior of $\triangle ABC$

Special lines in triangles

Special lines in triangles are lines (actually line segments) in triangles that have special characteristics. The special lines in this triangle are between the height, bisector, weight, and axis lines.

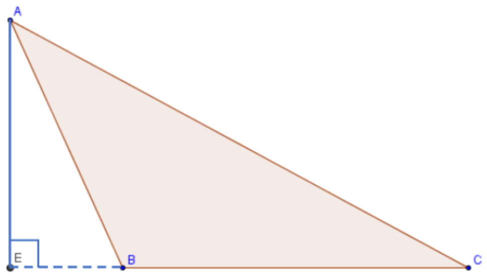
The altitude of the triangle

The *altitude of the triangle* is the line segment that connects the vertices of the triangle with the projection of that point on the side (or extension of the side) opposite it. In other words, an elevation is a line segment drawn from a vertex of a triangle perpendicular to the side (or extension of the side) opposite it. Each triangle has 3 concurrent height lines (all three intersect at one point).



$\overline{AD}$ high line $\triangle ABC$

$\overline{AD} \perp \overline{BC}$

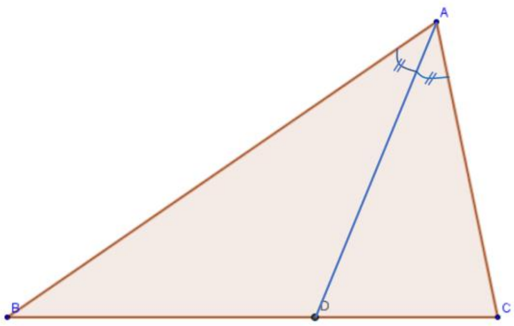


$\overline{AE}$ high line ΔABC

$\overline{AE} \perp$ extension $\overline{CB}$

Angle bisector of a triangle

A bisector of a triangle is a line segment that bisects the angles of a triangle that are congruent (congruent) and are connected to the opposite side. Each triangle has 3 concurrent bisectors (all three intersect at one point).

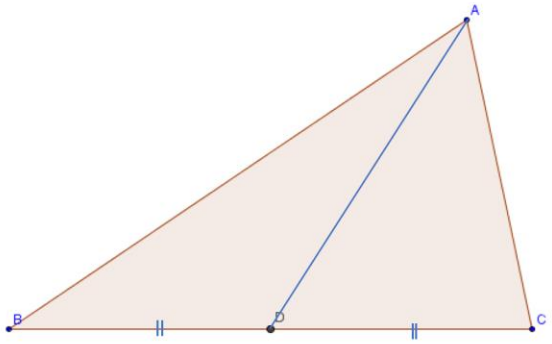


$\overline{AD}$ dividing line $\triangle ABC$

Then $\angle BAD \cong \angle DAC$

Medians

A median is a line segment that joins the vertex of an angle of a triangle with the midpoint of the opposite side. Each triangle has 3 concurrent lines of gravity (all three intersect at one point).

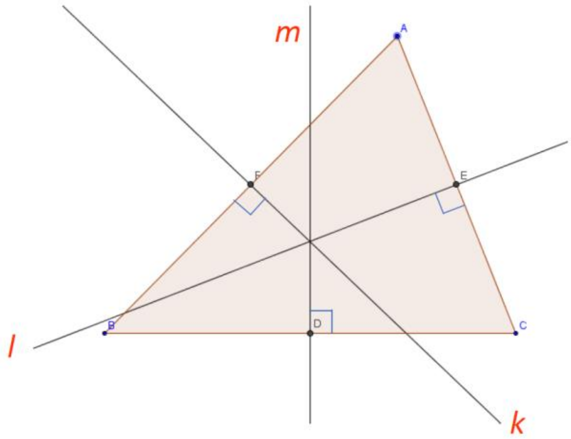


$\overline{AD}$ median $\triangle ABC$

Then $\overline{BD} \cong \overline{DC}$

Perpendicular bisector

The perpendicular bisector is the axis of the triangle is the line that passes through the midpoint of the side of the triangle and is perpendicular to it, each triangle has 3 concurrent axes (they intersect at one point).



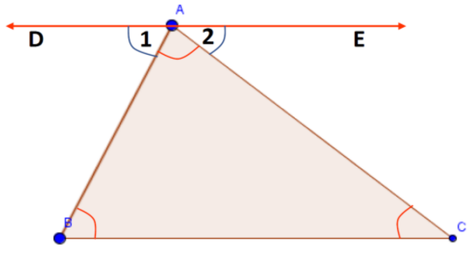
k, *l*, and *m* axis line $\triangle ABC$ then $\overline{BD} \cong \overline{DC}$; $\overline{CE} \cong \overline{EA}$; and $\overline{AF} \cong \overline{FB}$

$k \perp \overline{AB}$; $l \perp \overline{AC}$ and $m \perp \overline{BC}$

Several theorems on the triangle

Theorems; on scalene triangle $\triangle ABC$, that $m\angle A + m\angle B + m\angle C = 180^o$

Evidence:



Constructible $\overleftrightarrow{DE} \parallel \overline{BC}$ [through a point outside the line a line can be drawn parallel to the given line-postulate 10]

Obtained ∠1 ≅ ∠B dan ∠2 ≅ ∠C [If 2 parallel lines are transversed, then the opposite interior angles are congruent]

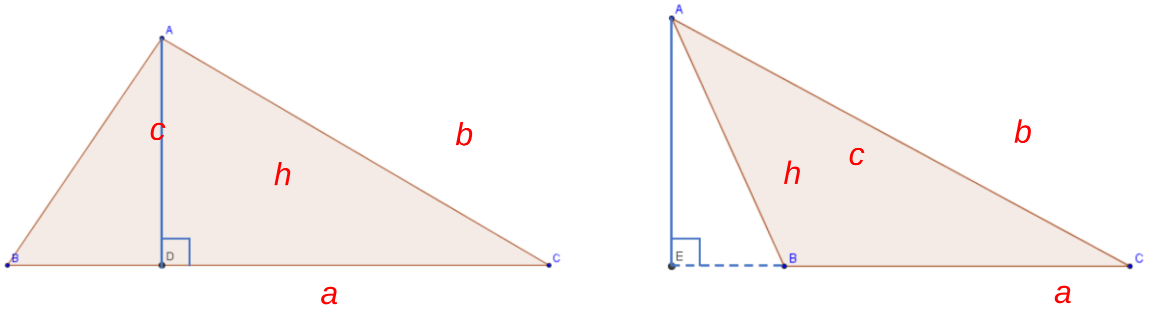
Then, m∠1 = m∠B and m∠2 = m∠C [definition of 2 congruent angles]

Whereas m∠1+ m∠A+ m∠2 = 180^o [∠DAE straight angle]

m∠B+ m∠A+ m∠C = 180^o [substitution]

proven;

Area of Triangle



Area △ABC = $\frac{1}{2}ah$

Perimeter △ABC = $a + b + c$

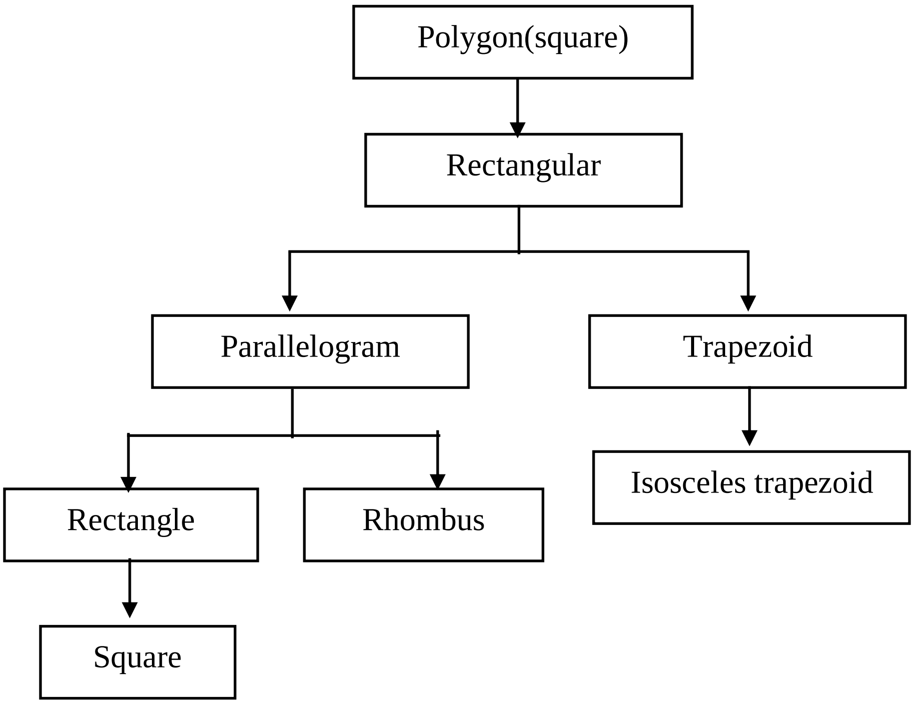
Pythagorean Theorem

In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the sides"

Proof: The proof of the Pythagorean Theorem is quite a lot and one of them is an exercise for the reader, namely in Problem 3

Square

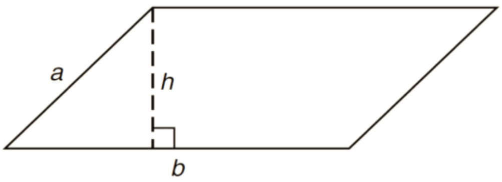
A square is a polygon that has four sides. The flow in the following diagram shows the definition of basic square geometry objects.



Definition of a quadrilateral, formula for area and perimeter

a. parallelogram (parallelogram) is a quadrilateral with opposite sides parallel

Take a look at the parallelogram sketch below!

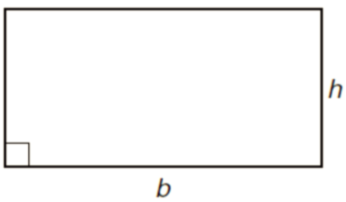


Perimeter formula (K) and area (L) parallelogram is

$K = 2a + 2b$ and $L = bh$

b. *The rectangle* is a parallelogram in which one of the angles is a right angle

Take a look at the rectangle sketch below!

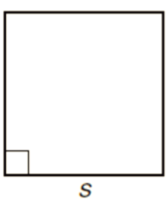


Perimeter formula (K) and area (L) rectangle is

$K = 2b + 2h$ and $L = bh$

c. *A Square* is a rectangle with two congruent sides

Take a look at the rectangle sketch below!

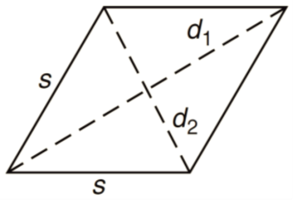


Perimeter formula (K) and area (L) of a square is

$K = 4s$ and $L = s^2$

d. *The rhombus* is a parallelogram with two congruent sides

Take a look at the rhombus sketch below!

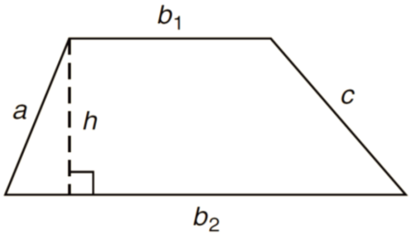


Perimeter formula (K) and area (L) of a rhombus is

$K = 4s$ and $L = \frac{1}{2}d_1 d_2$

e. *A trapezoid* is a quadrilateral that has one and only one pair of parallel sides

Take a look at the trapezoid sketch below!



Perimeter formula (K) and area (L) trapezoid is

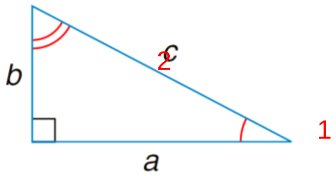
$K = a + b_1 + c + b_2$ and $L = \frac{1}{2}h(b_1 + b_2)$

Exercise

Problem 3

Answer and complete the following Pythagorean proofs using the concept of the relationship between angles and areas

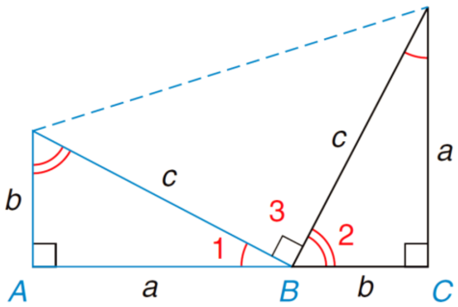
Step 1



Given a triangle with sides a, b, and c as shown above. What is the relationship between ∠1 dan ∠2? (Are they complementary or complementary?)

Give reasons or mathematical arguments for your answers!

Step 2



Next, by transforming the triangle as shown above (points A, B, and C are collinear), then prove that ∠3 is a right angle!

Step 3

By looking at the construction in Step 2, a right-angled trapezoid is formed. Next, using the concept of the area of a trapezoid, complete the following points!

Trapezoid areas $= \frac{1}{2}h(b_1 + b_2)$

$= \frac{1}{2}(\ldots + \cdots)(a + b)$

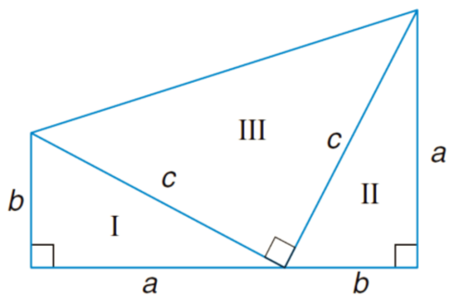
$= \frac{1}{2}(\ldots + b)^2$

$= \frac{1}{2}(a^2 + \ldots + b^2)$

$= \frac{1}{2}a^2 + \cdots + \cdots$

Step 4

The construction in Step 2 is restated with the following sketch/construction. Next, using the concept of the area of a triangle, complete the following points!



Trapezoid areas = areas △I + areas △II + areas △III

$= \frac{1}{2}\, ab + \ldots + \ldots$

$= ab + \ldots$

Step 5

The area of the Trapezoid in Step 3 is the same as the Area of the Trapezoid in Step 4, so we get

$\frac{1}{2}a^{2} + \cdots + \cdots = ab + \ldots$

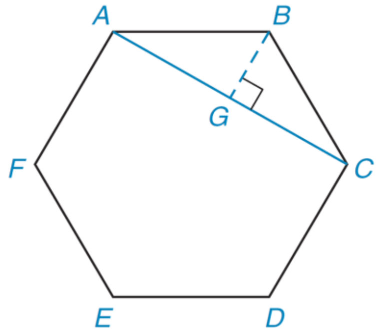
$$\frac{1}{2}a^{2} + \cdots = \cdots$$

$$a^2 + b^2 = c^2$$

Proven

Problem 4

Given a regular hexagon ABCDEF as shown below. If the length of the side of the hexagon is 6 cm and AC is diagonal, then find the area of the side AFEDC.

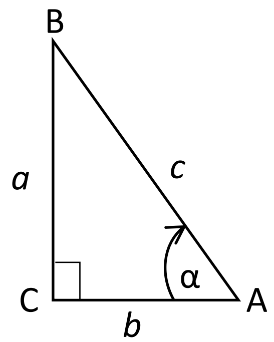


CHAPTER X
TRIGONOMETRY

Trigonometry is the ratio sides' value in a scalene triangle and right triangle. As for the sixth ratios in trigonometry including sine (sin α), cosine (cos α), tangent (tan α), cosecant (cosec α), secant (sec α) and cotangent (cot α). In this discourse, will more discuss right triangle, especially the elements in it that link to the trigonometry comparison.

A. Trigonometric Comparison in Certain Angle in Right Triangle

The besides image is a right triangle with the angle's corner point at C. The lengthy side in front of angle A is *a*, the lengthy side in front of angle B is *b*, and the lengthy side in front of angle C is *c*.



Look at angle α!

The side *a* is called right-angle side in front of angle α or can also called projector

The side *b* is called right-angle side near (along with) angle α or can also called projection

The c side (the hypotenuse) is called the hypotenuse or also known as the projectum

According to the information, 6 (six) trigonometric ratios to angle are defined as follows.

1. $$\sin \alpha = \frac{\text{length right - angle side in front of angle A}}{\text{lenght hypotenuse}} = \frac{a}{c}$$

2. $$\cos \alpha = \frac{\text{length right - angle side near (along with) angled A}}{\text{length hypotenuse}} = \frac{b}{c}$$

3. $$\tan \alpha = \frac{\text{length right - angle side in front of angle A}}{\text{length right - angle side near angle A}} = \frac{a}{b}$$

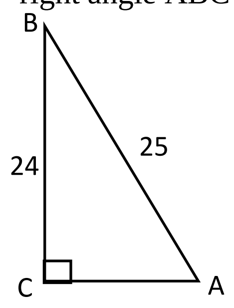
4. $$\csc \alpha = \frac{\text{length hypotenuse}}{\text{length right - angle side in front of angle A}} = \frac{c}{a}$$

5. $$\sec \alpha = \frac{\text{length hypotenuse}}{\text{length right - angle side near angle A}} = \frac{c}{b}$$

6. $$\cot \alpha = \frac{\text{length right - angle near angle A}}{\text{length right - angle side in front of angle A}} = \frac{c}{a}$$

Example 1.

In the following image right angle ABC with lengthy *a*= 24 and *c*= 25.



Determine the trigonometric ratios for sin α, cos α, and tan α function.

<u>Solution:</u>

The value of b is calculated by the pythagoras theorem

$b = \sqrt{25^2 - 24^2}$

$= \sqrt{625 - 576}$

$= \sqrt{49} = 7$

$\sin \alpha = \frac{a}{c} = \frac{24}{25}$

$\cos \alpha = \frac{b}{c} = \frac{7}{25}$

$\tan \alpha = \frac{a}{b} = \frac{24}{7}$

B. Trigonometric Comparison Value for Special Angles

Special angle is angle which the trigonometric ratios' can be found without using mathematic table or calculator, that is angle 0°, 30°, 45°,60°, and 90°. The special angles that will be learned are 30°, 45°, and 60°.

To find the value of the trigonometric ratios of special angles, a right triangle is used as shown below.

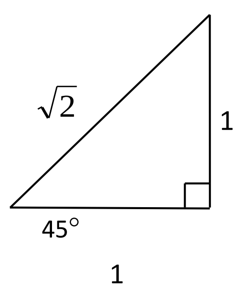


Image 1.a. Special Angle

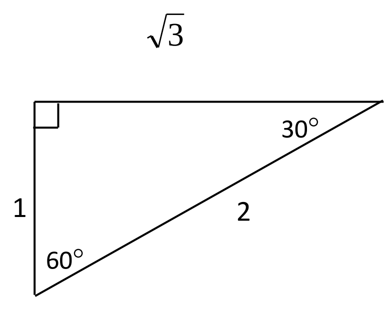


Image 1.b. Special Angle

From image 1.a can be determined:

$$\sin 45° = \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{2}$$

$$\cos 45° = \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{2}$$

$$\tan 45° = \frac{1}{1} = 1$$

Next, from image 1.b can be determined:

$$\sin 30° = \frac{1}{2} \qquad \sin 60° = \frac{\sqrt{3}}{2} = \frac{1}{2}\sqrt{3}$$

$$\cos 30° = \frac{\sqrt{3}}{2} = \frac{1}{2}\sqrt{3} \qquad \cos 60° = \frac{1}{2}$$

$$\tan 30° = \frac{1}{\sqrt{3}} = \frac{1}{3}\sqrt{3} \qquad \tan 60° = \frac{\sqrt{3}}{1} = \sqrt{3}$$

Based on these calculations, the table of trigonometric comparison values for special angles is presented in the following table.

Trigonometric Ratios	*Special Angles*				
	0^0	$\mathbf{30^0}$	$\mathbf{45^0}$	$\mathbf{60^0}$	$\mathbf{90^0}$
		$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	
		$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	
		$\frac{1}{3}\sqrt{3}$		$\sqrt{3}$	undefined

Example 2.

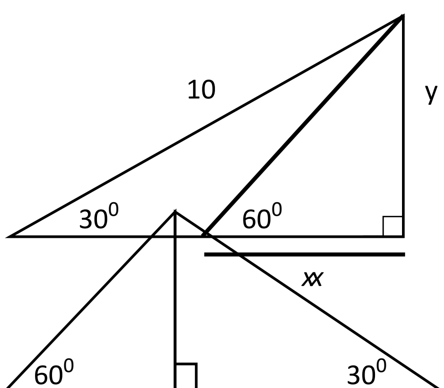
1. $\sin 30° + \cos 45° = \frac{1}{2} + \frac{1}{2}\sqrt{2} = \frac{1+\sqrt{2}}{2}$

2. $\sin 45° \tan 60° + \cos 45° \cot 60° = \frac{1}{2}\sqrt{2} \cdot \sqrt{3} + \frac{1}{2}\sqrt{2} \cdot \frac{1}{3}\sqrt{3}$

$$= \frac{1}{2}\sqrt{6} + \frac{1}{6}\sqrt{6} = \frac{4}{6}\sqrt{6} = \frac{2}{3}\sqrt{6}$$

3. Calculate the value of x in the following figure.

a.



b.

Solution:

a. $\sin \alpha = \frac{y}{10}$

$\sin 30° = \frac{y}{10}$

$$\frac{1}{2} = \frac{y}{10}$$

$2y = 10 \rightarrow y = 5$

Next $\tan 60° = \frac{y}{x}$

$\sqrt{3} = \frac{5}{x}$

$x = \frac{5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$

b. $\tan \alpha = \frac{y}{2}$

$\tan 60° = \frac{y}{2}$

$\sqrt{3} = \frac{y}{2}$

$y = 2\sqrt{3}$

Next $\sin 60° = \frac{y}{x}$

$\sqrt{3} = \frac{2\sqrt{3}}{x}$

$x = 2$

C. Trigonometric Comparison of an Angle in Different Quadrant

P is scalene dot in the Quadrant I with coordinates (x,y). OP is line which can rotate about the origin O in Cartesian coordinates, so that $\angle XOP$ can be 0° up to 90°. Need to know that

$OP = \sqrt{x^2 + y^2} = r$ dan $r > 0$

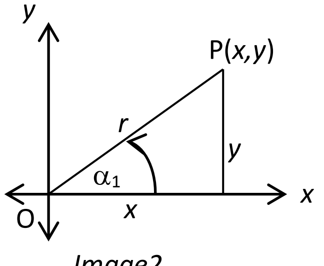


Image2.

According to the Image , the three standard trigonometric ratios can be defined in terms of abscissa (x), ordinate (y), and length of OP (r) as follows:

1. $$\sin \alpha = \frac{\text{ordinate P}}{\text{length OP}} = \frac{y}{r}$$

2. $$\cos \alpha = \frac{\text{abscissa P}}{\text{length OP}} = \frac{x}{r}$$

3. $$\tan \alpha = \frac{\text{ordinate P}}{\text{abscissa P}} = \frac{y}{x}$$

By rotating the OP line so ∠XOP = α can be located in quadrant I, quadrant II, quadrant III or quadrant IV, as shown in Image 3. Follows:

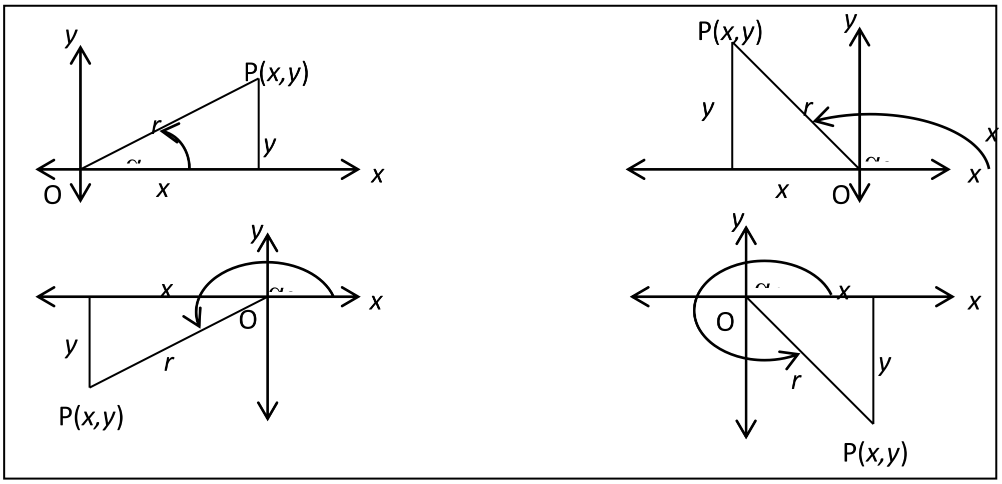


Image 3. Dot in different quadrants

The table of values for the three trigonometric comparisons in each quadrant is as follows.

Trigonometric Ratios Quadrant

D. Relating Angle Trigonometric Comparison Formula

The angles related to angle α are angles (90°±α), (180°±α), (360°±α), and -α°. Two related angles have special names, for example, the **angler** (complement) for the angle α° with (90° - α) and **straightener** (supplement) for angle α° with (180° - α).

Example 3.

Angler angle 50° is 40° and straightener angle 110° is 70°.

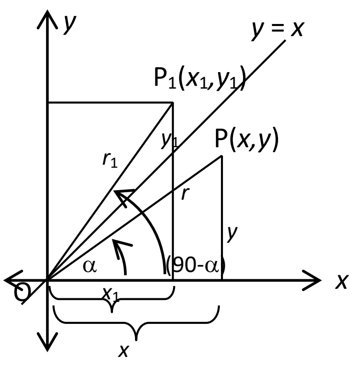
1) <u>Trigonometric ratio for angle α with (90° - α)</u>

Based to the image 4. Besides, is known

Dot $P_1(x_1,y_1)$ is the shade of $P(x,y)$ due to line's reflection $y=x$, so can be obtained:

a. ∠XOP = α and $\angle XOP_1 = 90° - \alpha$

b. $x_1 = x$, $y_1 = y$ and $r_1 = r$



by using those correlation, so can be obtained:

a. $\sin(90° - \alpha) = \frac{y_1}{r_1} = \frac{x}{r} = \cos\alpha$

b. $\cos(90° - \alpha) = \frac{x_1}{r_1} = \frac{y}{r} = \sin\alpha$

c. $\tan(90° - \alpha) = \frac{y_1}{x_1} = \frac{x}{y} = \cot\alpha$

Based on those calculations, the trigonometric comparison formula of α with (90° - α) can be written as follows.

a.	$\sin(90° - \alpha) = \cos\alpha$	d. $\text{cosec}(90° - \alpha) = \sec\alpha$
b.	$\cos(90° - \alpha) = \sin\alpha$	e. $\sec(90° - \alpha) = \text{cosec}\,\alpha$
c.	$\tan(90° - \alpha) = \cot\alpha$	f. $\cot(90° - \alpha) = \tan\alpha$

2) Trigonometric ratio for angle α° with (180° - α)

Dot $P_1(x_1,y_1)$ in Image 5. Besides is shade of $P(x,y)$ due to the y-axis' reflection, so

a. $\angle XOP = \alpha$ and $\angle XOP_1 = 180° - \alpha$

b. $x_1 = -x$, $y_1 = y$ and $r_1 = r$

than can be obtained the correlation:

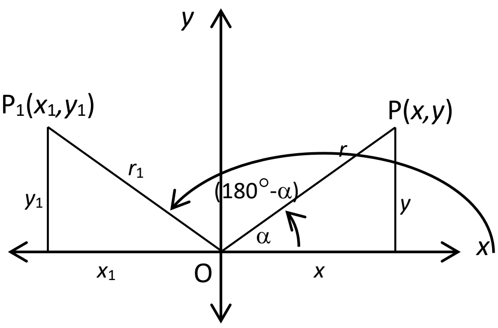


Image 6. Related angle

a. $\sin(180° - \alpha) = \frac{y_1}{r_1} = \frac{y}{r} = \sin\alpha$

b. $\cos(180° - \alpha) = \frac{x_1}{r_1} = \frac{-x}{r} = -\cos\alpha$

c. $\tan(180° - \alpha) = \frac{y_1}{x_1} = \frac{y}{-x} = -\tan\alpha$

According to those correlation, so the formula can be obtained:

a.	$\sin(180° - \alpha) = \sin\alpha$	d. $\text{cosec}(180° - \alpha) = \text{cosec}\,\alpha$
b.	$\cos(180° - \alpha) = -\cos\alpha$	e. $\sec(180° - \alpha) = -\sec\alpha$
c.	$\tan(180° - \alpha) = -\tan\alpha$	f. $\cot(180° - \alpha) = -\cot\alpha$

3) Trigonometric ratio for angle $\alpha°$ with $(180° + \alpha)$

Dot $P_1(x_1,y_1)$ in the image 6. Beside is the shade from dot $P(x,y)$ due to the line $y=-x$ reflection, so

a. $\angle XOP = \alpha$ and $\angle XOP_1 = 180° + \alpha$

b. $x_1 = -x$, $y_1 = -y$ and $r_1 = r$

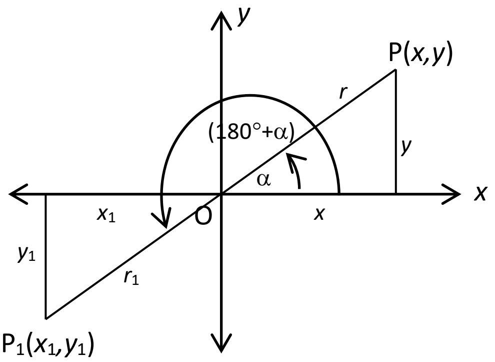


Image 6. The related angle

then the correlation can be obtain:

a. $$\sin(180° + \alpha) = \frac{y_1}{r_1} = \frac{-y}{r} = -\sin\alpha$$

b. $$\cos(180° + \alpha) = \frac{x_1}{r_1} = \frac{-x}{r} = -\cos\alpha$$

c. $$\tan(180° + \alpha) = \frac{y_1}{x_1} = \frac{-y}{-x} = \frac{y}{x} = \tan\alpha$$

According to those correlation, so formula can be obtained:

a.	$\sin(180° + \alpha) = -\sin\alpha$	d. $\text{cosec}(180° + \alpha) = -\text{cosec}\,\alpha$
b.	$\cos(180° + \alpha) = -\cos\alpha$	e. $\sec(180° + \alpha) = \sec\alpha$
c.	$\tan(180° + \alpha) = \tan\alpha$	f. $\cot(180° + \alpha) = \cot\alpha$

4) Trigonometric for angle α with $(-\alpha)$

Dot $P_1(x_1,y_1)$ in the Image 7. Beside is the shade from $P(x,y)$ due to the reflection to axis-x, so

a. $\angle XOP = \alpha$ and $\angle XOP_1 = -\alpha$

b. $x_1 = x$, $y_1 = -y$ and $r_1 = r$

then can be obtained the correlation:

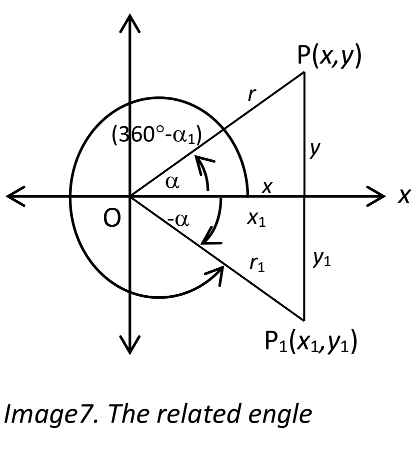


Image7. The related engle

a. $$\sin(-\alpha) = \frac{y_1}{r_1} = \frac{-y}{r} = -\sin\alpha$$

b. $$\cos(-\alpha) = \frac{x_1}{r_1} = \frac{x}{r} = \cos\alpha$$

c. $$\tan(-\alpha) = \frac{y_1}{x_1} = \frac{-y}{x} = -\tan\alpha$$

According to those correlation, the formula can be obtained:

a.	$\sin(-\alpha) = -\sin\alpha$	d. $\text{cosec}(-\alpha) = -\text{cosec}\,\alpha$
b.	$\cos(-\alpha) = \cos\alpha$	e. $\sec(-\alpha) = \sec\alpha$

c. $\tan(-\alpha) = -\tan\alpha$ f. $\cot(-\alpha) = -\cot\alpha$

Whereas the ratio formula for angle $(360° - \alpha)$ and α identical to the negative angle formula, for example $\sin(360°-\alpha) = -\sin\alpha$

Example 4.

1. Express the following trigonometric ratios into the trigonometric ratios of the complement angles.

a. sin 27° d. cot 88°

b. cos 46° e. sec 64°

c. tan 12° f. cosec 75°

2. Express the following trigonometric ratios into the trigonometric ratios of straightener angles.

a. sin 138° d. cot 146°

b. cos 99° e. sec 162°

c. tan 117° f. cosec 123°

3. Express the following trigonometric ratios into the trigonometric ratios of acute angle.

a. sin 234° d. cot 212°

b. cos 312° e. sec 354°

c. tan 199° f. cosec 284°

Solution:

1. The complement angle from:

a. sin 27° = sin (90° - 63°) = cos 63°

b. cos 46° = cos (90° - 44°) = sin 44°

c. tan 12° = tan (90° - 78°) = cot 78°

d. cot 88° = cot (90° - 2°) = tan 2°

e. sec 64° = sec (90° - 26°) = cosec 26°

f. cosec 75° = cosec (90° - 15°) = sec 15°

2. The straightener angle from:

a. sin 138° = sin (180° - 42°) = sin 42°

b. cos 99° = cos (180° - 81°) = -cos 81°

c. tan 117° = tan (180° - 63°) = -tan 63°

d. cot 146° = cot (180° - 34°) = -cot 34°

e. sec 162° = sec (180° - 18°) = -sec 18°

f. cosec 123° = cosec (180° - 57°) = cosec 57°

3. The acute angle from:

a. sin 234° = sin (180° + 54°) = -sin 54°

b. cos 312° = cos (360° - 48°) = cos 48°

c. tan 199° = tan (180° + 19°) = tan 19°

d. cot 212° = cot (180° + 32°) = -cot 32°

e. sec 354° = sec (360° - 6°) = sec 6°

f. cosec 284° = cosec (360° - 76°) = -cosec 76°

E. Trigonometric Comparison Formula for angle (*n*.360° - α°) and angle (*n*.360° + α°)

Look at the image below!

In image 8. to presuppose ∠ POX = α° and ∠ QOX = (*n*.360° - α°) with *n* is Z (integers) that affect dot Q which is in the angle that has value (-α°).

Thus, relation formulas for comparison for angle (*n*.360° - α°) identical with negative angle (-α), for example sin (*n*.360°–α°) =-sin α°

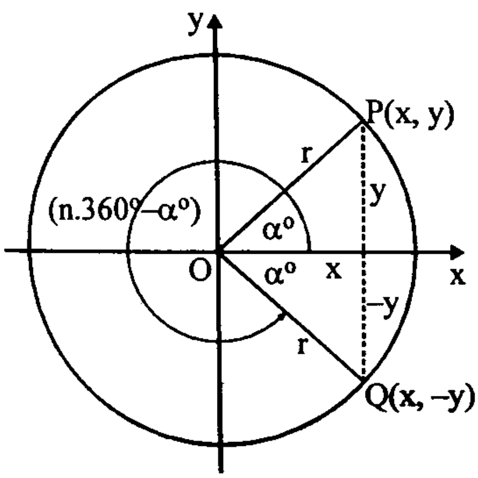


Image8.

According to those collocation, the formula can be obtained:

a. sin (*n*.360°–α°) =sin (- α°) = -sin α°

b. cos (*n*.360°–α°) =cos (- α°) = cosα°

c. tan (*n*.360°–α°) =tan (- α°) = -tanα°

d. cot (*n*.360°–α°) =cot (- α°) = -cotα°

e. sec (*n*.360°–α°) =sec (- α°) = secα°

f. cosec (*n*.360°–α°) =cosec (- α°) = -cosecα°

next, look at the image 9. beside!

In that image, ∠ POX = α° and ∠ QOX = (n.360° +α°) with n is Z (integers) that affect dot Q coincide with dot P.

Hance, the ratio formulas' for angle (n.360° +α°) = formulas trigonometric ratio for angle (α°).

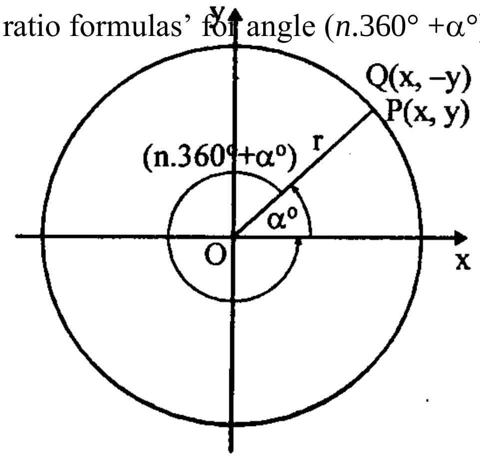


Image 9.

So based on those collocations, the formula can be obtained:

a. sin (n.360°+α°) = sin α°
b. cos (n.360°+α°) =cos α°
c. tan (n.360°+α°) =tan α°
d. cot (n.360°+α°) =cot α°
e. sec (n.360°+α°) =sec α°
f. cosec (n.360°+α°) =cosec α°

Example 5.

1. Express the following trigonometric ratios into acute angle trigonometric ratios.

a. sin 400°
b. cos 385°
c. tan 518°
d. cot 437°
e. sec 622°
f. cosec 594°
g. sin 710°
h. sin 872°
i. cos 926°
j. cos 1.025°
k. tan 1.369°
l. tan 1.215°

2. Calculate the value of the following trigonometric ratios.

a. sin 420°
b. cos 450°
c. tan 390°
d. sin 510°
e. cos 585°
g. sin 630°
h. sin 750°
i. cos 990°
j. cos 1.155°
k. tan 1.485°
l. tan 2.550°

f. tan 480°

Solution:

sin 400°		sin (1.360° + 40°)
		sin 40°
cos 385°		cos (1.360° + 25°)
		cos 25°
tan 518°		tan (1.360° + 158°)
		tan 158°
		tan (180° – 22°)
		tan 22°
cot 437°		sin (2.360° – 283°)
		-sin 283°
		-sin (360° – 77°)
		sin 77°
sec 622°	=	sec (2.360° – 98°)
	=	-sec 98°
	=	-sec (180° – 82°)
	=	sec 82°
cosec 594°	=	cosec (1.360 + 234°)
	=	cosec 234°
	=	cosec (180° + 54°)
	=	-cosec 54°
sin 710°	=	sin (2.360° – 10°)
	=	-sin 10°
sin 872°	=	sin (2.360° + 152°)
	=	sin 152°
	=	sin (180° – 28°)
	=	sin 28°
cos 926°	=	cos (3.360° – 154°)
	=	-cos 154°
	=	-cos (180° – 26°)
	=	cos 26°
cos 1.025°	=	cos (3.360° – 55°)
	=	-cos 55°

$$\tan 1.369° = \tan (3.360° + 289°)$$
$$= \tan 289°$$
$$= \tan (360° – 71°)$$
$$= -\tan 71°$$
$$\tan 1.215° = \tan (4.360° – 225°)$$
$$= -\tan 225°$$
$$= -\tan (180° + 45°)$$
$$= -\tan 45°$$
$$\sin 420° = \sin (1.360° + 60°)$$
$$= \sin 60°$$
$$= \frac{1}{2}\sqrt{3}$$
$$\cos 450° = \cos (1.360° + 90°)$$
$$= \cos 90°$$
$$= 0$$
$$\tan 390° = \tan (1.360° + 30°)$$
$$= \tan 30°$$
$$= \frac{1}{3}\sqrt{3}$$
$$\sin 510° = \sin (2.360° – 210°)$$
$$= -\sin 210°$$
$$= -\sin (180° + 30°)$$
$$= \sin 30°$$
$$= \frac{1}{2}$$
$$\cos 585° = \cos (2.360° – 135°)$$
$$= -\cos 135°$$
$$= -\cos (180° – 45°)$$
$$= \cos 45°$$
$$= \frac{1}{2}\sqrt{2}$$
$$\tan 480° = \tan (1.360 + 120°)$$
$$= \tan 120°$$
$$= \tan (180° – 60°)$$
$$= -\tan 60°$$
$$= -\sqrt{3}$$

sin 630°	=	sin (2.360° – 90°)
	=	-sin 90°
	=	-1
sin 750°	=	sin (2.360° + 30°)
	=	sin 30°
	=	$\frac{1}{2}$
cos 1.035°	=	cos (3.360° – 45°)
	=	-cos 45°
	=	$-\frac{1}{2}\sqrt{2}$
cos 1.125°	=	cos (3.360° + 45°)
	=	cos 45°
	=	$\frac{1}{2}\sqrt{2}$
tan 1.395°	=	tan (4.360° – 45°)
	=	-tan 45°
	=	-1
tan 2.550°	=	tan (7.360° + 30°)
	=	tan 30°
	=	$\frac{1}{3}\sqrt{3}$

PRACTICE

1) Express the following trigonometric ratios into acute angle trigonometric ratios.

a) cos 687°

b) tan 830°

c) cot 1.327°

d) sec 992°

e) cosec 718°

g) sin 1.589°

h) sin 2.706°

i) cos 1.227°

j) cos 1.104°

k) tan 2.011°

l) tan 1.201°

2) Simplify the following form.

a) $\frac{\cos (90° - \alpha°)}{\cos (90° + \alpha°)}$

b) $\frac{\sin (180° - \alpha°)}{\sin (90° - \alpha°)}$

Prove or shows that:

3) cos 60° + sin 60° tan 60° = 2

4) $\frac{\tan 60° - \tan 30°}{1 + \tan 60° \cdot \tan 30°} = \tan 30°$

5) Known that ∆ KLM right-angle at M. The length of the based side 6 and sin ∠ LKM = $\frac{1}{2}$. Define:

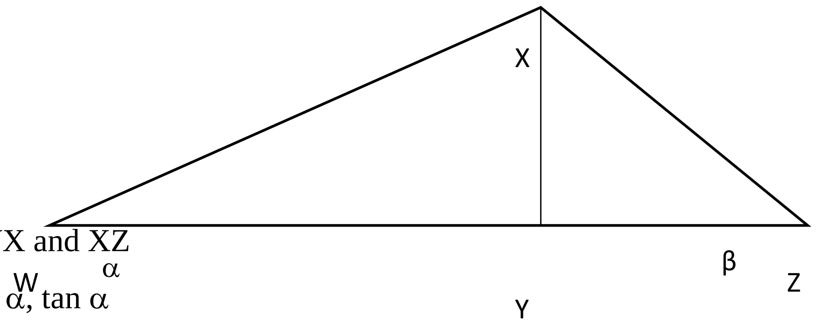
a) The length another both sides

b) sin ∠ KLM

c) cos ∠ LKM

d) cos ∠ KLM

6) Known that ∆ XYZ is right-angle isosceles. Length WZ = 34 and XY = 10.



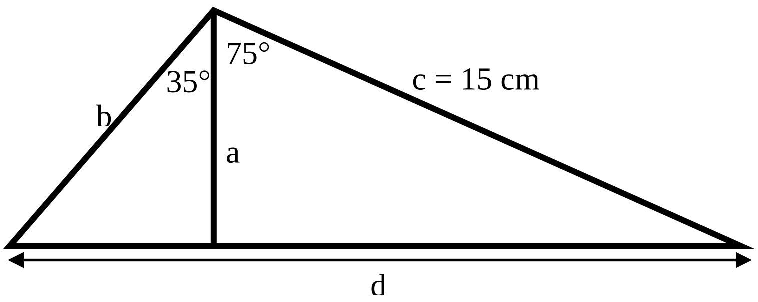
Define:

a) Length WX and XZ

b) sin α, cos α, tan α

c) sin β, cos β, tan β

Look at the following image! Calculate:



a) Length a!

b) Length b!

c) Length d!

(accurate to 3 decimal places)

7) Known that sin γ° = $-\frac{2}{3}$ and tan γ° positive. Define cos γ°, sec γ°, cosec γ°, and cot γ°!

8) Known that tan θ° = $-\frac{3}{7}$ and cos θ° negative. Define sin θ°, cot θ°, sec θ°, and cosec θ°!

9) Known that cos β° = $-\frac{2}{3}$, calculate sin α°, tan α°, cot α°, sec α°, and cosec α°!

CHAPTER XI

LIMIT

This chapter discusses the concept of limits consisting of an algebraic function and trigonometric function.

Limits of Algebraic Function

Limit value in x = a

Definition:

The function $\boldsymbol{f(x)}$ has a limit L for x towards a (a is called a limit point), i.e $\lim\limits_{x\to a} \boldsymbol{f(x) = L}$, if for each $\boldsymbol{\varepsilon} > 0$ there is such $\boldsymbol{\delta} > 0$ that if $\boldsymbol{0} < |\boldsymbol{x - a}| < \delta$ applies:

$$|\boldsymbol{f(x) - L}| < \varepsilon$$

Definition above does not mention the value of $\boldsymbol{f(x)}$ in $\boldsymbol{x = a}$. So, for $\lim\limits_{x\to a} \boldsymbol{f(x)}$ to exist, $\boldsymbol{f(x)}$ does not have to be defined in di $\boldsymbol{x = a}$.

For example: $\lim\limits_{x\to 1} \frac{x^2+2x-3}{x-1} = \mathbf{4},$ but the function $\boldsymbol{f(x)} = \frac{x^2+2x-3}{x-1}$ is undefined in $\boldsymbol{x = 1}$

If f(a) is defined, limit value $\lim\limits_{x\to a} f(x) = f(a)$

Example 1

Find the value $\lim\limits_{x\to 3} \frac{x^2+3x}{x+2}$

Answer:

$$\lim_{x\to 3} \frac{x^2+3x}{x+2} = \frac{3^2+3\,.\,3}{3+2} = \frac{9+9}{5} = \frac{18}{5}$$

If f(a) $= \frac{0}{0}$ (indeterminate form), the value of $\lim\limits_{x\to a} f(x)$ is solved by:

1) Factoring the numerator and denominator of f(x) with a factor (x – a) so that it can be simplified

2) Multiplying the numerator and denominator by the sequence when there is a root shape, then simplified

3) Specifying the form of numerator and denominator derivatives so that an indeterminate value is obtained (not 0/0)

Infinite Limit (∞)

The basic formula $\lim_{x\to\infty}\frac{1}{x^n}=0$ for n positif numbers

a. $$\lim_{x\to\infty}\frac{a_nx^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0}{b_mx^m+a_{m-1}x^{m-1}+\cdots+b_1x+b_0}=\begin{cases}\infty \ if\ n>m\\ \frac{a_n}{b_m}\ \ if\ n=m\\ 0\ if\ n<m\end{cases}$$

b. $$\lim_{x\to\infty}\left(\sqrt{f(x)}-\sqrt{g(x)}\right)$$

The value is determined by multiplying $\lim_{x\to\infty}\left(\sqrt{f(x)}-\sqrt{g(x)}\right)$ with $\left(\frac{\sqrt{f(x)}+\sqrt{g(x)}}{\sqrt{f(x)}+\sqrt{g(x)}}\right)$, so the form that can be simplified is obtained, then can be acquired a certain value (not $\infty-\infty$)

The Properties of Limits of Function

a. $\lim_{x\to c}k=k$

b. $\lim_{x\to c}k\times f(x)=k\times\lim_{x\to c}f(x)$

c. $\lim_{x\to c}f(x)\pm g(x)=\lim_{x\to c}f(x)\pm\lim_{x\to c}\ g(x)$

d. $\lim_{x\to c}f(x)\times g(x)=\lim_{x\to c}f(x)\times\lim_{x\to c}\ g(x)$

e. $\lim_{x\to c}\frac{f(x)}{g(x)}=\frac{\lim_{x\to c}\ f(x)}{\lim_{x\to c}\ g(x)}$, provided that g(x) ≠0

f. $\lim_{x\to c}[f(x)]^n=\left[\lim_{x\to c}f(x)\right]^n$

g. $\lim_{x\to c}\sqrt[n]{f(x)}=\sqrt[n]{\lim_{x\to c}f(x)}$, provided that $\lim_{x\to c}f(x)\geq 0$ for n even numbers.

h. $\lim_{x\to\infty}\frac{k}{x^n}=0$

Limits of Trigonometric Functions

1. The Concept of Trigonometric Limits

Look at some of the following function limits.

$\lim_{x\to 2\pi}\frac{\boldsymbol{x^2}-2}{\boldsymbol{x}}$	$\lim_{x\to\pi}\frac{x}{\sin x}$
$\lim_{x\to 0}\sin x$	$lim_{x\to 0}\frac{\tan x}{x}$
$\lim_{x\to\pi}\tan x$	$\lim_{x\to 0}\frac{1-\cos x}{x}$
$\lim_{x\to\frac{\pi}{2}}\cos x$	$\lim_{x\to\frac{\pi}{2}}\sin x\cos x$
$\cos x\lim_{x\to\pi}\mathrm{x}^2$	$\lim_{x\to\frac{\pi}{3}}\frac{\cos x}{\tan x}$
$\sin 2x\lim_{x\to\pi}\mathrm{x}$	

Some of limits above can be devided into two groups as follows.

a. The group of trigonometric function limits

$\lim\limits_{x\to 0} \sin x$, $\lim\limits_{x\to \pi} \tan x$, $\lim\limits_{x\to \frac{\pi}{2}} \cos x$, $\lim\limits_{x\to \pi} \frac{x}{\sin x}$, $\lim\limits_{x\to 0} \frac{\tan x}{x}$, $\lim\limits_{x\to 0} \frac{1-\cos x}{x}$, $\lim\limits_{x\to \frac{\pi}{2}} \sin x \cos x$, and $\lim\limits_{x\to \frac{\pi}{3}} \frac{\cos x}{\tan x}$ is the limit of trigonometric function.

b. The group of non trigonometric function limit

$\lim\limits_{x\to 2\pi} \frac{\boldsymbol{x^2-2}}{\boldsymbol{x}}$, $\cos x \lim\limits_{x\to \pi} x^2$, and $\sin 2x \lim\limits_{x\to \pi} x$ is not the limit of trigonometric function.

> The limit of a trigonometric function contains a trigonometric function as a function subjected to a limit operation.

How to determine the value of a trigonometric function limit?

Consider the following example

Example 2

Determine the value $\lim\limits_{x\to 0} \sin x$

Answer:

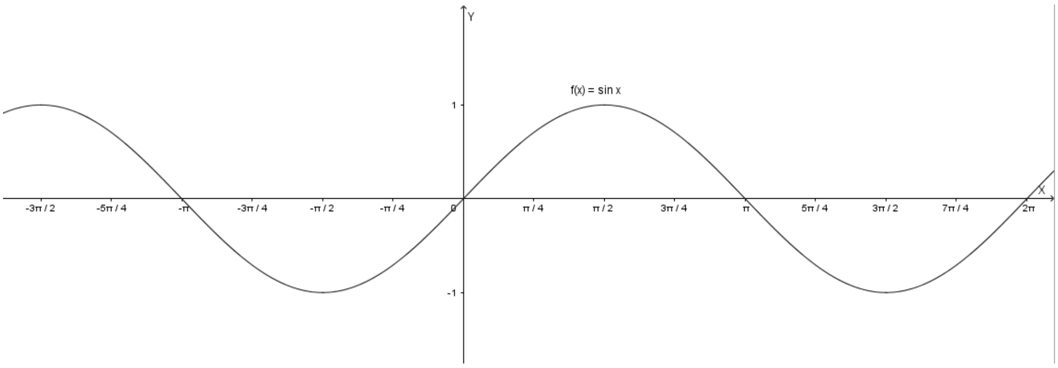
To answer it, you can use tables and graphs. Consider the table of values of the $f(x) = \sin x$ function as well as following graph.

Values table of the function $f(x) = \sin x$ for x in radians.

			1	[illegible]		
sin x	4	8	1	[illegible]	3	4

Approach from the left → ← Approach from the right

Function graph $f(x) = \sin x$.



From the tables and graphs of the function $f(x) = \sin x$, can be concluded:

a. For an x value close to 0 from the left, the sin x value is close to 0. Thus, obtained the mathematical notation $\lim_{x\to 0^-} \sin x = 0$.

b. For an x value close to 0 from the right, the value of sin x is close to 0. Thus, obtained the mathematical notation $\lim_{x\to 0^+} \sin x = 0$.

We know, $\lim_{x\to 0^-} \sin x = 0$ and $\lim_{x\to 0^+} \sin x = 0$ it can be concluded that $\lim_{x\to 0} \sin x = 0$.

2. Finding the Limit Properties of Trigonometric Function

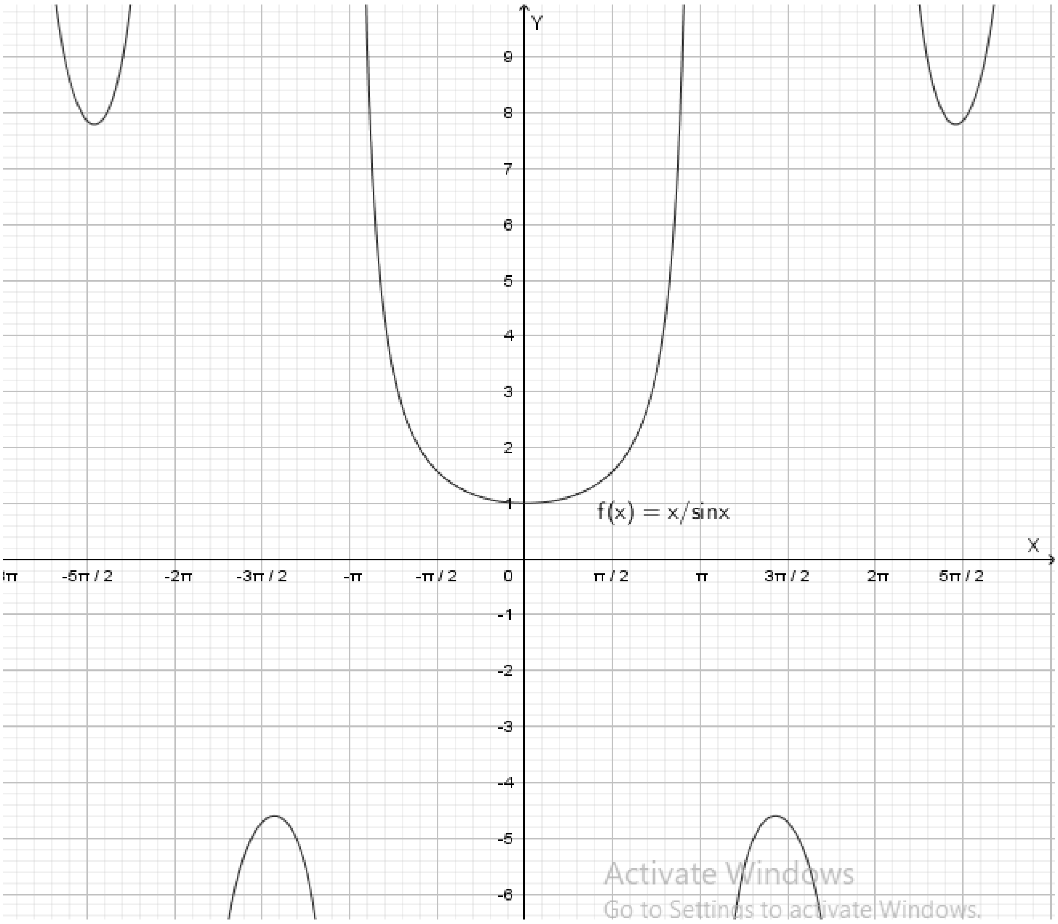
a. The property of $\lim_{x\to 0} \frac{x}{\sin x} = 1$ and $\lim_{x\to 0} \frac{\sin x}{x} = 1$

After determining the $\lim_{x\to 0} \sin x$, you will learn and show that the values of $\lim_{x\to 0} \frac{x}{\sin x} = 1$ and $\lim_{x\to 0} \frac{\sin x}{x} = 1$. To demonstrate it, look at the table of values for $f(x) = \frac{x}{\sin x}$ bellow.

Table of values for $f(x) = \frac{x}{\sin x}$

Approach from the left → ← Approach from the right

in x											
$f(x) = \frac{x}{\sin x}$		7	.7						.7	7	



Function Graph $f(x) = \frac{x}{\sin x}$

From the table and graph above, it can be seen that $\lim_{x \to 0^-} \frac{x}{\sin x} = 1$ and $\lim_{x \to 0^+} \frac{x}{\sin x} = 1$. Thus, it can be concluded that $\lim_{x \to 0} \frac{x}{\sin x} = 1$

b. The property of $\lim_{x \to 0} \frac{x}{\tan x} = 1$ and $\lim_{x \to 0} \frac{\tan x}{x} = 1$

In addition to the proprerties of $\lim_{x \to 0} \frac{x}{\sin x} = 1$ and $\lim_{x \to 0} \frac{\sin x}{x} = 1$, other properties that apply to the limits of trigonometric functions are $\lim_{x \to 0} \frac{x}{\tan x} = 1$ and $\lim_{x \to 0} \frac{\tan x}{x} = 1$

3. Determining the Limit Value of a Trigonometric Function

a. Substitution method

Direct substitution means substituting the value of x, for example x = c, into the form of a function. The function value for that value of x = c is the value of $\lim_{x \to c} f(x)$.

Not all limit values of trigonometric functions can be determined by this way (direct substitution). The value of x = c must meet certain conditions so that $\lim_{x \to c} f(x)$ can be determined in this way.

Example 3

Determine the value $\lim_{x\to\frac{\pi}{3}}\frac{\sin 2x}{\cos x}$

Answer:

Domain of a function $f(x)=\frac{\sin 2x}{\cos x}$ is $\left\{x\middle|\cos x\neq 0 \Leftrightarrow x\neq \pm\frac{\pi}{2}+k\,.2\pi\right\}$ so $x=\frac{\pi}{3}$ is a member of the function domain $f(x)=\frac{\sin 2x}{\cos x}$

$$\lim_{x\to\frac{\pi}{3}}\frac{\sin 2x}{\cos x}=\frac{\sin 2(\frac{\pi}{3})}{\cos\frac{\pi}{3}}=\frac{\sin\frac{2\pi}{3}}{\cos\frac{\pi}{3}}=\frac{\frac{1}{2}\sqrt{3}}{\frac{1}{2}}=\sqrt{3}$$

So, the value of $\lim_{x\to\frac{\pi}{3}}\frac{\sin 2x}{\cos x}=\sqrt{3}$

Example 4

Determine the value $\lim_{x\to\frac{\pi}{3}}\frac{\sin 2x}{\cos\frac{3}{2}x}$

Answer:

Domain of a function $f(x)=\frac{\sin 2x}{\cos\frac{3}{2}x}$ is $\left\{x\middle|\cos\frac{3}{2}x\neq 0 \Leftrightarrow x\neq \pm\frac{\pi}{3}+k\,.2\pi\right\}$ so $x=\frac{\pi}{3}$ is not a member of the function domain $f(x)=\frac{\sin 2x}{\cos\frac{3}{2}x}$

$$\lim_{x\to\frac{\pi}{3}}\frac{\sin 2x}{\cos\frac{3}{2}x}=\frac{\sin 2(\frac{\pi}{3})}{\cos\frac{3}{2}.\frac{\pi}{3}}=\frac{\sin\frac{2\pi}{3}}{\cos\frac{\pi}{2}}=\frac{\frac{1}{2}\sqrt{3}}{0}=undefined$$

So, the value of $\lim_{x\to\frac{\pi}{3}}\frac{\sin 2x}{\cos\frac{3}{2}x}=undefined.$

Conclusion:

x = c is a member of the function domain *f(x).*

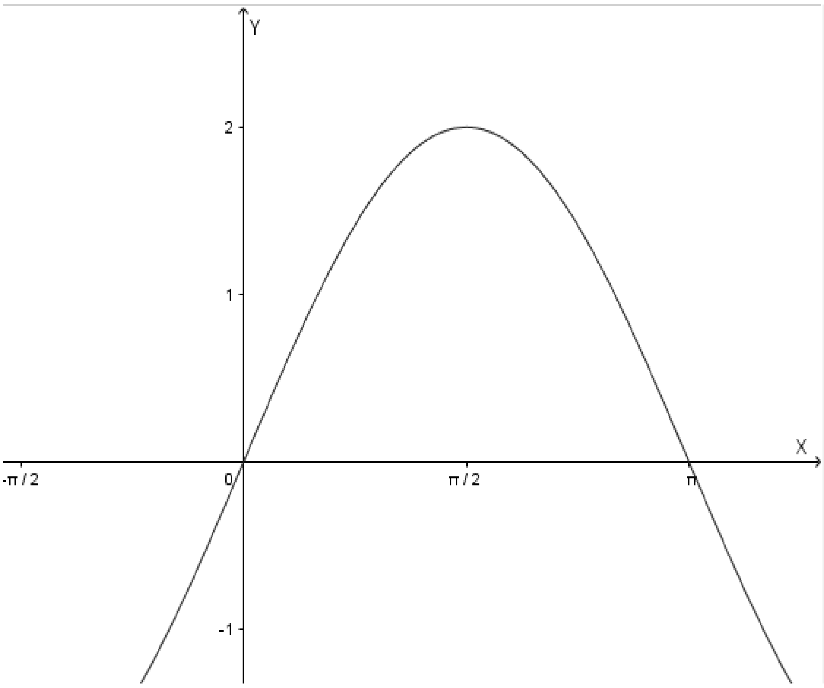
b. Faktorization method

Look at $\lim_{x\to\frac{\pi}{3}}\frac{\sin 2x}{\cos x}$. What if the value of x approached changes, for example $x\to\frac{\pi}{2}$? What is the value of $\lim_{x\to\frac{\pi}{2}}\frac{\sin 2x}{\cos x}$?

By direct substitution, obtained:

$\lim_{x\to\frac{\pi}{2}}\frac{\sin 2x}{\cos x}=\frac{\sin 2(\frac{\pi}{2})}{\cos\frac{\pi}{2}}=\frac{\sin\pi}{\cos\frac{\pi}{2}}=\frac{0}{0}\leftarrow -$ indeterminate form

The value $\lim_{x\to\frac{\pi}{2}}\frac{\sin 2x}{\cos x}=\frac{0}{0}$ does not mean that the limit value does not exist. If a graph of $f(x)=\frac{\sin 2x}{\cos x}$ is drawn, the value of the function for x approaches $\frac{\pi}{2}$ from the left and for x approaches $\frac{\pi}{2}$ from the right is a real number. The number is the same, which is a number very close to 2. It means, the value of $\lim_{x\to\frac{\pi}{2}} \frac{\sin 2x}{\cos x}$ exists and is a real number. Look at the following graph $f(x) = \frac{\sin 2x}{\cos x}$.



Now it will be shown that $\lim_{x\to\frac{\pi}{2}} \frac{\sin 2x}{\cos x} = 2.$

Pay attention to the following steps.

$$\lim_{x\to\frac{\pi}{2}} \frac{\sin 2x}{\cos x} = \lim_{x\to\frac{\pi}{2}} \frac{2\ \sin x \cos x}{\cos x}$$

$$= \lim_{x\to\frac{\pi}{2}} 2 \sin x$$

$$= 2 \sin \frac{\pi}{2} = 2 \,.\, 1 = 2$$

Conclusion:

If a limit has an indeterminate value, the form of the function subjected to the limit operation must be changed to another form first, one of which is by factoring so that the same factor can be eliminated

c. Using Trigonometric Limit Properties

If a function has an indeterminate form while not being able to be factored, it can take advantage of the limit properties of trigonometry. Consider the example below.

Example 5

Determine the value $\lim_{x\to 0} \frac{\cos 3x - \cos x}{x^2}$

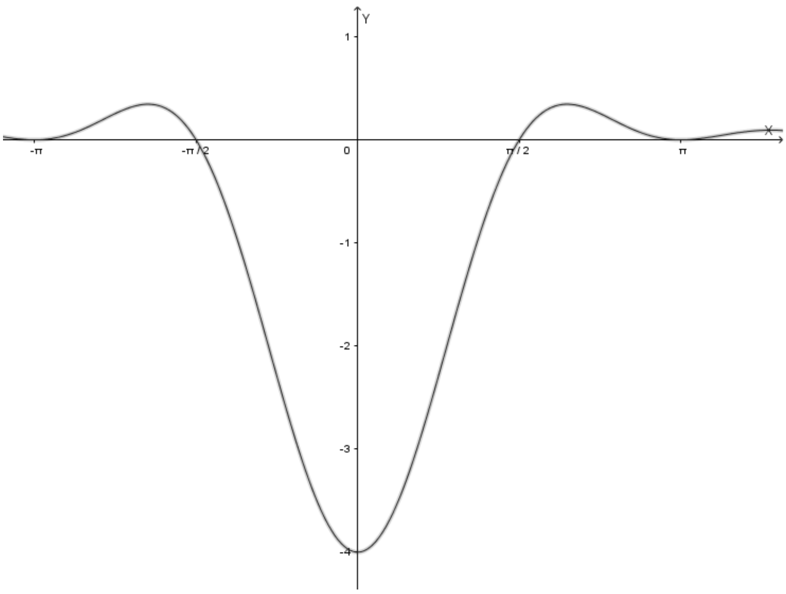
Answer:

By direct substitution, we get:

$\lim_{x \to 0} \frac{\cos 3x - \cos x}{x^2} = \frac{\cos 0 - \cos 0}{0} = \frac{1-1}{0} = \frac{0}{0} \leftarrow -$ indeterminate form

Graphically the function $f(x) = \frac{\cos 3x - \cos x}{x^2}$ shows the real number for x close to 0 from the left as well as from the right.

Look at the following graph.



From the graph, it can be seen that the value of f(x) for x is close to 0 from the left or from the right is close to -4.

Now it will be shown how to find the value of $\lim_{x \to 0} \frac{\cos 3x - \cos x}{x^2}$

$$\lim_{x \to 0} \frac{\cos 3x - \cos x}{x^2} = \lim_{x \to 0} \frac{-2 \sin\left(\frac{3x + x}{2}\right) \sin\left(\frac{3x - x}{2}\right)}{x^2}$$

$$= \lim_{x \to 0} \frac{-2 \sin 2x \sin x}{x^2}$$

$$= \lim_{x \to 0} \frac{-2 \,.\, 2 \sin x \cos x \sin x}{x^2}$$

$$= \lim_{x \to 0} \frac{-4 \sin x \cos x \sin x}{x^2}$$

$$= \lim_{x \to 0} -4 \cos x \frac{\sin x \sin x}{x \,.\, x}$$

$$= \lim_{x \to 0} -4 \cos x \,.\frac{\sin x}{x} \,.\, \frac{\sin x}{x}$$

$$= -4 \,.\lim_{x \to 0} \cos x \,.\frac{\sin x}{x} \,.\, \frac{\sin x}{x}$$

$$= -4 \,.\lim_{x \to 0} \cos 0 \,.\, 1 \,.\, 1$$

$$= -4\,.\,1\ .1\,.\,1$$

$= -4$

Thus, the value of $\lim_{x\to 0}\frac{\cos 3x-\cos x}{x^2} = -4$

> **Cunclusion:**
>
> If the trigonometric function is indeterminate and cannot be factored, you must change the shape of the function so that it contains the form $\frac{x}{\sin x}, \frac{\sin x}{x}, \frac{x}{\tan x}$ or $\frac{\tan x}{x}$. If the function already contains these forms, the limit properties of the trigonometric function can be used to determine the limit value. The limit properties of such trigonometric function are $\lim_{x\to 0}\frac{x}{\sin x} = 1,\ \lim_{x\to 0}\frac{\sin x}{x} = 1, \lim_{x\to 0}\frac{x}{\tan x} = 1,$ and $\lim_{x\to 0}\frac{\tan x}{x} = 1$

EXERCISE

1. The value of $\lim_{x\to -3}\frac{\sqrt{x+12}-3}{x^2+7x+12} = \cdots$
2. If $n = \lim_{x\to 4}\frac{x^2-16}{3-\sqrt{x+5}}$, the value of *n* = . . .
3. The value of $\lim_{x\to 0}\frac{3x+4x^{-1}}{4x-x^{-1}} = \cdots$
4. The value of $\lim_{x\to\infty}\left(x+2-\sqrt{x^2+3x-4}\right) = \cdots$
5. The value of $\lim_{x\to\infty}\left(\sqrt{25x^2+x-5}-5x-2\right) = \cdots$
6. Determine the value of $\lim_{x\to 0}\frac{x^2+\sin^2 x}{1-\cos 2x}$
7. The value of $\lim_{x\to 0}\frac{3x+4x^{-1}}{4x-x^{-1}} = . . .$
8. The value of $\lim_{x\to 2}\frac{(x^2-3)-(3-x)}{(x-2)\left(\sqrt{x^2-3}+\sqrt{3-x}\right)} = . . .$
9. The value of $\lim_{x\to\infty}\left(\sqrt{25x^2+x-5}-5x-2\right) = \cdots$
10. The value of $\lim_{x\to\infty}\left(\sqrt{25x^2-9x-16}-5x+3\right) = \cdots$
11. The value of $\lim_{x\to -3}\frac{\tan(x^2-9)}{x+3} = \cdots$
12. The value of $\lim_{x\to\frac{\pi}{3}}\frac{\tan(3x-\pi)\cos 2x}{\sin(3x-\pi)} = \cdots$
13. The value of $\lim_{x\to 0}\frac{\cos x-\cos 3x}{1-\cos 2x} = \cdots$

14. The value of $\lim_{x \to -3} \frac{x^2+2x-3}{(x-3)\sin(x+3)} = . .$

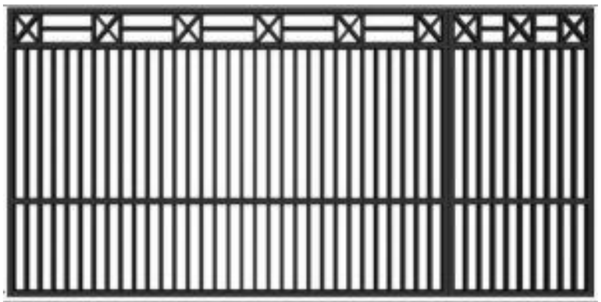
15. The value of $\lim_{x \to 0} \frac{3x-\sin 2x}{4x-\tan 3x} = \cdots$

16. The value of $\lim_{x \to \frac{\pi}{2}} \frac{2-\sin 2x}{6x-3\pi} = \cdots$

CHAFTER 12
DERIVATIVES OF FUNCTIONS

This chapter discusses the concept of derivatives of functions consisting of: derivatives as gradients of tangents, derivatives of algebraic functions, and applications of derivatives.

Look at the following picture



Source:https://www.aluminiumkacadepok.com/wp-content/uploads/2019/01/Pagar-Iron Hollow-Galvanis-300x151.jpg

If someone plans to make a gate frame and has galvanized iron with a total length of 14 meters, what is the maximum area of the gate frame that can be made with the iron? To answer this question, it is necessary to recall the concepts of the perimeter and area of a rectangle as a form of the gate frame, namely the perimeter $K = 2(p + l)$ formula and the area formula $L = p \times l$. In the case above, it is known that the material we have is 14 meters of galvanized iron which will be made into a rectangular gate frame. in other words we have the perimeter of the rectangle 14m. The formula for the perimeter of a rectangle is $K = 2(p + l) = 14m$. From this perimeter formula we operate

$2(p + l) = 14$

$p + l = \frac{14}{2}$

$l = 7 - p$

l

. The next value we substitute into the rectangular area formula.

$L = p \times l$

l

. The next value we substitute into the rectangular area formula.

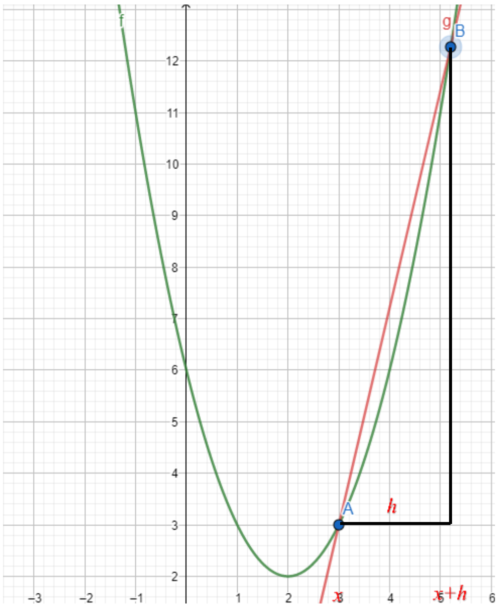
$L = p \times l$

$L = p \times (7 - p)$

$L \quad = 7p - p^2$

We get an area function with the independent variable . To determine the maximum area value of the area function, it is necessary to use the derivative concept which will be studied in this discussion. This discussion consists of derivatives of functions as tangent gradients, derivatives of algebraic functions and derivative applications.

A. DERIVATIVES OF FUNCTIONS AS GRADIENTS OF A tangent line

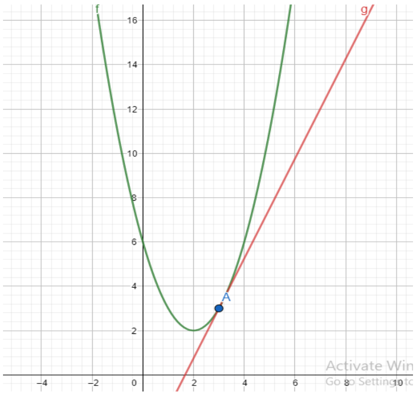


Suppose we have a graph of $f(x)$ a continuous function. Then there is a line that intersects the function at the point $A(x, f(x))$ and point $B(x+h, f(x+h))$. Then the slope of the line can be written.

$$M_{AB} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x+h) - f(x)}{x + h - x} = \frac{f(x+h) - f(x)}{h}$$

If we move point A and point B until they are close to each other, or in other words until the distance h approaches the value 0, what will happen?

What happens is that line AB, which had cut the curve $f(x)$ at two points, will eventually intersect the curve at one point.



By using the concept of limit, the gradient of the line remains the same as before and can be written

$$M_{AB} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

If there is an ility, then this value M_{AB} is called the first derivative of the function..M_{AB} can be written with a derived symbol, namely $f'(x)$ (read: *f* accent *x*).

Example A.1

Suppose we have a function $f(x) = x^2 - 3x + 4$, determine the gradient of the tangent to the function at the point $x = 2$!

To answer this question, we can use the concept of derivatives above:

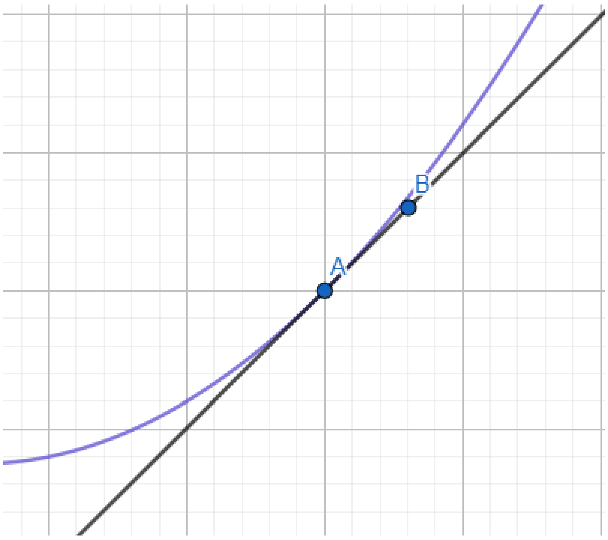
It is known $f(x) = x^2 - 3x + 4$, so $f(2)$ dan $f(2+h)$ written

$$f(2) \quad = 2^2 - 3.2 + 4$$
$$= 2$$
$$f(2+h) \quad = (2+h)^2 - 3(2+h) + 4$$
$$= h^2 + 4h + 4 - 6 - 3h + 4$$
$$= h^2 + h + 2$$

so that the gradient of the tangent to the function at the point can be found using the concept of derivative

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} \frac{h^2 + h + 2 - (2)}{h}$$
$$= \lim_{h \to 0} \frac{h^2 + h}{h}$$
$$= \lim_{h \to 0} h + 1$$
$$= \lim_{h \to 0} h + \lim_{h \to 0} 1$$
$$= 0 + 1$$
$$= 1$$

So, the gradient of the tangent to the function $f(x) = x^2 - 3x + 4$ di titik $x = 2$ at the point is 1.



Example A.2

Determine the derivative of the function $f(x) = x^2 + 5x - 6$!

Given $f(x) = x^2 + 5x - 6$, then $f(x)$ and $f(x + h)$ can be written

$f(x) \quad = x^2 + 5x - 6$

$f(x + h) \quad = (x + h)^2 + 5(x + h) - 6$

$= x^2 + 2xh + h^2 + 5x + 5h - 6$

The derivative of the function $f(x)$ can be written

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x^2 + 2xh + h^2 + 5x + 5h - 6) - (x^2 + 5x - 6)}{h}$$

$$= \lim_{h \to 0} \frac{(2xh + h^2 + 5h)}{h}$$

$$= \lim_{h \to 0} 2x + h + 5$$

$$= 2x + 5$$

So the derivative of the function $f(x) = x^2 + 5x - 6$ is $f'(x) = 2x + 5$. This means that to find out the tangent to the function $(x) = x^2 + 5x - 6$ at the point $x = 3$, we just need to substitute the value $x = 3$ equation $f'(x) = 2x + 5$ the $f'(3) = 2.3 + 5 = 11$ yang berarti gradient garis singgung fungsi adalah 11.

A. DIRECTIONS OF ALGEBRA FUNCTIONS

1. Constant function The constant

function can be written as $f(x) = a$ which means the gradient of the tangent to the function is 11.

$f(x) = a$ dan $f(x + h) = a$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{a - a}{h}$$

$$= \lim_{h \to 0} \frac{0}{h}$$

$$= \lim_{h \to 0} 0$$

$$= 0$$

So, the derivative of a constant function $f(x) = a$ is $f^{'}(x) = 0$

2. Linear function A linear

function can be written as $f(x) = ax + b$, whwre a and b is any real number. The derivative of a linear function can be determined as follows. $f(x) = ax + b$ and $f(x + h) = a(x + h) + b = ax + ah + b$

$$f^{'}(x) \quad = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{ax + ah + b - (ax + b)}{h}$$

$$= \lim_{h \to 0} \frac{ah}{h}$$

$$= \lim_{h \to 0} a$$

$$= a$$

So, the derivative of a constant function $f(x) = ax + b$ is $f'(x) = a$

2. Linear function A linear

function can be written as v$f(x) = ax^2 + bx + c$, where a,b and c is any real number. The

derivative of a linear function can be determined as follows. $f(x) = ax^2 + bx + c$ dan $f(x + h) = a(x + h)^2 + b(x + h) + c = ax^2 + ah^2 + 2axh + bx + bh + c$

$$f^{'}(x) \quad = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(ax^2 + ah^2 + 2axh + bx + bh + c) - (ax^2 + bx + c)}{h}$$

$$= \lim_{h \to 0} \frac{ah^2 + 2axh + bh}{h}$$

$$= \lim_{h \to 0} ah + 2ax + b$$

$$= 2ax + b$$

So, the derivative of a constant function $f(x) = ax^2 + bx + c$ is $f^{'}(x) = 2ax +$ 3.polynomial functon $f(x) = ax^n$

fungsi polynomial $f(x) = ax^n$ function can be written as.

$f(x) = ax^n$, $f(x + h)$ can be found using Newton's Binomial formula..

$$
\begin{aligned}
f(x+h) &= a(x+h)^n \\
&= a(x^n + nx^{n-1}h + C_2^n x^n h^2 + \cdots + h^n) \\
&= ax^n + anx^{n-1}h + aC_2^n x^n h^2 + \cdots + ah^n
\end{aligned}
$$

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0} \frac{(ax^n + anx^{n-1}h + aC_2^n x^n h^2 + \cdots + ah^n) - (ax^n)}{h} \\
&= \lim_{h\to 0} \frac{(anx^{n-1}h + aC_2^n x^n h^2 + \cdots + ah^n)}{h} \\
&= \lim_{h\to 0} \frac{h(anx^{n-1} + aC_2^n x^n h + \cdots + ah^{n-1})}{h} \\
&= \lim_{h\to 0} anx^{n-1} + aC_2^n x^n h + \cdots + ah^{n-1} \\
&= anx^{n-1}
\end{aligned}
$$

So the derivative of the function $f(x) = ax^n$ is $f'(x) = anx^{n-1}$.

Example B.1

Determine the derivative of the function $f(x) = 3x^4 - 2x^3 + 2x^2 - 5x + 8$!

Solution

Using the derivative rules for polynomial functions, the $f(x)$ above function can be solved as follows. $f(x) = 3x^4 - 2x^3 + 2x^2 - 5x + 8$

$$= 4.3x^{4-1} - 3.2x^{3-1} + 2.2x^{2-1} - 1.5x^{1-1} + 0$$

$$= 12x^3 - 6x^2 + 4x^1 - 5$$

THEOREM

Suppose there is a function $f(x)$ and $g(x)$

1. For example, $h(x) = f(x) \pm g(x)$, then the derivative of the function $h(x)$ can be written as follows. $h'(x) = f'(x) \pm g'(x)$

2. For examle $h(x) = f(x).g(x)$, then the derivative of the function $h(x)$ can be written as follows $h'(x) = f'(x)g(x) + g'(x)f(x)$

3. For example $h(x) = \frac{f(x)}{g(x)}$, then the derivative of the function $h(x)$ can be written as follows.

4. $h'(x) = \frac{f'(x)g(x) - g'(x)f(x)}{(g(x))^2}$

Example B.2

1. *Determine the derivative of the following function!* $f(x) = (2x^3 + 3x)^2$

$$g(x) = \frac{3x}{2x-1}$$

Solution

Function $f(x) = (2x^3 + 3x)^2$ can be written as $f(x) = (2x^3 + 3x)(2x^3 + 3x)$. By using the two product theorem, the derivative of the function $f(x)$ can be solved as follows.

For Examle $h(x) = 2x^3 + 3x$, than $h^{'}(x) = 3.2x^{3-1} + 3 = 6x^2 + 3$

$$f^{'}(x) = h'(x)g(x) + g'(x)g(x)$$
$$= (6x^2 + 3)(2x^3 + 3x) + (6x^2 + 3)(2x^3 + 3x)$$
$$= 2(6x^2 + 3)(2x^3 + 3x)$$

2. Th Funcion $g(x) = \frac{3x}{2x-1}$ consists of two functions namely $f(x) = 3x$ and $h(x) = 2x - 1$. By using the two product theorem, the derivative of the function $g(x)$ can be solved as follows..

$$g^{'}(x) = \frac{f^{'}(x)h(x) - h^{'}(x)f(x)}{\left(h(x)\right)^2}$$

$f(x) = 3x,$ $\quad f^{'}(x) = 3$

$h(x) = 2x - 1$ $\quad h(x) = 2$

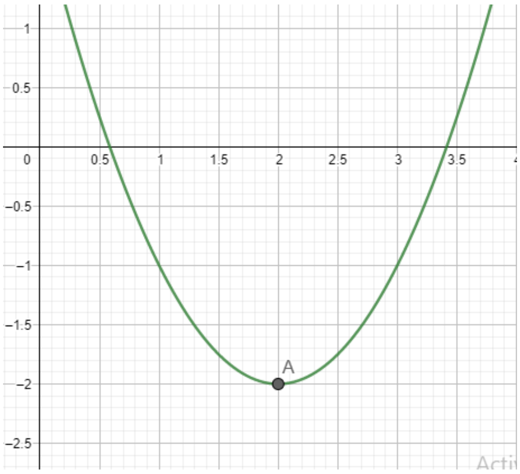
$$g^{'}(x) = \frac{f'(x)h(x) - h'(x)f(x)}{\left(h(x)\right)^2}$$

$$= \frac{3(2x-1) - 2(3x)}{(2x-1)^2}$$

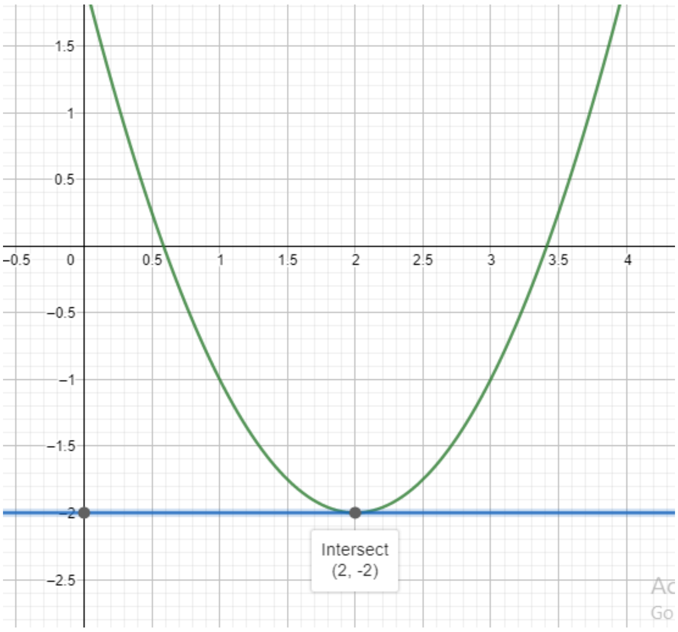
$$= \frac{6x - 3 - 6x}{(2x-1)^2} = \frac{-3}{(2x-1)^2}$$

B. DERIVATIVE APPLICATIONS

1. Extreme values (maximum and minimum values of a function)



The quadrad function $f(x) = x^2 - 4x + 2$ as in the picture above has a minimum extreme value, namely $(2, -2)$. As it is known that the minimum or maximum value of the quadratic function is obtained from $\left(-\frac{b}{2a}, \frac{D}{4a}\right)$. Note that it $f'(x)$ is a tangent gradient, so from the following graph it can be seen that the tangent at the extreme point is a line parallel to the axis x. It appears that the gradient of the tangent parallel to the axis x is equal to 0. 0 $m = 0$



example, in the above case, we will look for the gradient of the tangent line at the $(2, -2) f(x) = x^2 - 4x + 2$

$$f'(x) = 2x - 4$$

Untuk $x = 2$

$$f'(2) = 2^2 - 4 = 0$$

Dari kasus tersebut diperoleh bahwa nilai turunan pada titik minimum adalah $f'(x) = 0$.

Example C. If someone plans to make a gate frame and has galvanized iron with a total length of 14 meters, determine the maximum area of the gate frame that can be made with the iron!.

Solution

It is known that the perimeter of a rectangular gate is 14 meters.

The formula for the perimeter of a rectangle

$K = 2(p + l) = 14$

$p + l = 7$

$p = 7 - l$

The formula for the area of a rectangle

$L = p \times l$ substitution values p

$= (7 - l) \times l$

$= 7l - l^2$

The area function L will be maximized if $L'(x) = 0$

$L(x) \quad = 7l - l^2$

$L'(x) \quad = 7 - 2l \qquad = 0$

$l \qquad = \frac{7}{2} = 3{,}5$

So the maximum area of the gate

$L \qquad = 7l - l^2$

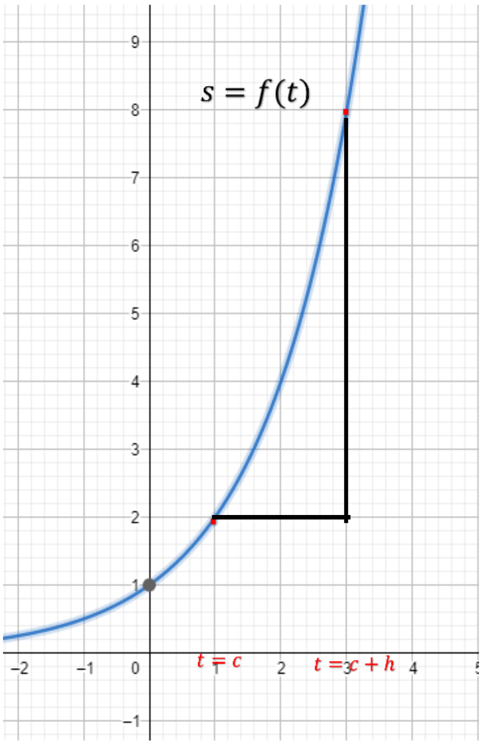
$= 7.(3{,}5) - (3{,}5)^2$

$= 7l - l^2$

$= 24{,}5 - 12{,}25$

$= 12{,}25$

So the maximum area of the gate that can be made is 12.25 meters. 2. Velocity



If $s = f(t)$ we express the equation of motion of an object along a straight line, where s is the displacement or direct distance of the object from the starting point in time t. fuction f that describes the motion is called the position function of the object. At intervals from $t = c$ up to $t = c + h$. position change is $f(c + h) - f(c)$. The average velocity in this time interval is

$$Kecepatan\ rata - rata = perpindahan\ waktu = \frac{f(c + h) - f(c)}{h}$$

Misalkan kita akan menghitung selang waktu yang sangat kecil (h mendekati 0), maka kita memperoleh yang namanya kecepatan sesaat untuk $t = c$.

$$= v(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$$

Suppose we will calculate a very small time interval $s(t) = f(t)$ close to t certain tim is :$v(t) = s'(t)$ or $v(t) = f\ '(t)$.

The rate of change of velocity is called acceleration. Following the definition of velocity above, acceleration can be defined as the derivative of velocity.

$a(t) = v'(t)$ where $a(t)$ is acceleration and $v(t)$ is velocity.

Example B.2

An object moves according to the function $s(t) = t^3 - 3t^2 + 2$, where it is safe to s state distance and t express time. Determine a

a Speed and acceleration in time !

b. The speed and acceleration of the object in 5 seconds!

c. When does that thing stop!

a. Solution

$$s(t) = t^3 - 3t^2 + 2$$

$$v(t) = s'^{(t)} = 3t^2 - 6t$$

$$a(t) = v'^{(t)} = 6t - 6$$

b. The object's speed in 5 seconds is

$$v(5) = 3.5^2 - 6.5 = 75 - 30 = 45\ m/s$$

the object's acceleration in 5 seconds

$$a(5) = 6.5 - 6 = 30 - 6 = 24\ m/s^2$$

The object will stop when its velocity is equal to 0.

$v(t) \quad = 0$

$3t^2 - 6t \quad = 0$

$3t(t2) \quad = 0$

$t = 0$ atau $t = 2$

So the object will stop at $t = 2\ detik$

a. EXERCISE

1. Determine the derivative of the following function by defining the derivative as the limit of the gradient of the tangent line!. $f(x) = 3x - 2$

b. $g(x) = x^2 - 2$

c. $f(x) = x^3 + 3x$

1. Determine the derivative of the following function by defining the derivative as the limit of the gradient of the tangent line!. $y = f(x) = 3x^2 - 3x + 1$ yang bergradien 15!

2. It is known that a line is tangent to the graph of the function $f(x) = x^2 + 2x - 3$ Determine the equation of the line! $y = -2x + 5$!

a. Determine the coordinates of the tangent point of the tangent $f(x) = x^5 - 3x^4 + 6x^3 - 3x + 8$

b. $f(x) = (3x^2 - 2)(2x + 3)$

c. $f(x) = \frac{x^2 - 4x}{2x}$

3. Pak Subur's garden is rectangular in shape with a circumference of 60 meters. If the length x meters and the width is y meters, determine

a. a. The equation that expresses the relationship between x and y!

b. The size of Pak Subur's garden for maximum area! 7. An object moves with the equation of motion $y = t^3 - 12t + 8$ in y meters and t in seconds Determine the speed and acceleration of the object in $t = 5$ seconds!

CHAPTER XIII
STATISTICS

This chapter discusses the size of data concentration which consists of: mean, median, mode
In everyday life, there are many situations where we will interact with certain data. In order to understand the data, we can use a number of statistical measures. These measures will help us to generalize a group of data, make conclusions, and compare it with other data sets. These statistical measures include *mean*, median, mode, *range*, deviation from the mean, and absolute deviation from the *mean*.

The mean, median, and mode are three general measures of central tendency; they are three frequently used mathematical tools to analyze data. *The mean*, commonly referred to as the mean, is the sum of all the datums divided by the number of data items. The median is the middle number in a data set ordered from smallest to largest. If the number of data values is even, we take the average of the two numbers in the middle to find the median. Finally, the mode is the number that occurs most often.

In this section, we will specifically discuss the concept of data centering measures, both for single data and grouped data. Each explanation is accompanied by example questions and their solutions to make it easier for readers to understand the concept as a whole. At the end of this chapter, we also include practice questions and answer keys.

A. The mean

*(*also known as the *mean*) often referred to as the average is one of the most common and frequently used measures of data centering. In simple terms, *mean* is the sum of all data components, divided by the number of data. In addition to the arithmetic *mean*, such as the geometric mean, *weighted* harmonic mean, and the mean. However, these three kinds of *mean* will not be discussed further in this section.

Before discussing further, the following is a histogram image that represents symmetrical and asymmetrical (skewed) data. These two images are intended to show how the characteristics of the data are based on their symmetry and the position *mean of* their respective

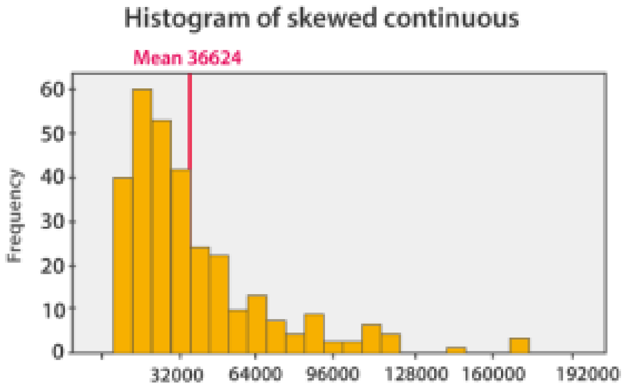


Figure 1. Histogram of asymmetrically distributed data (skew). Source: cuemath.com

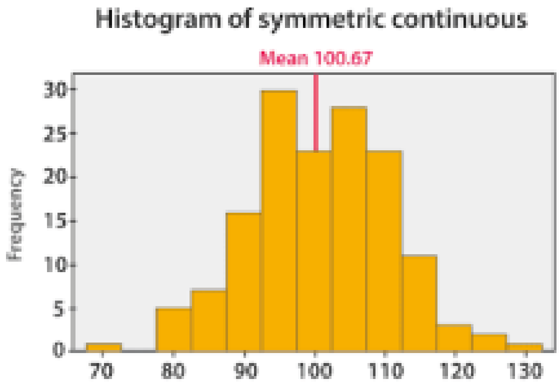


Figure 2. Histogram of normally distributed data. Source: cuemath.com

In symmetrical data, the *means* exactly in the middle, as in figure 2. However, in asymmetrical data, the extreme values located at the end (tail) of the histogram, cause the *mean* shift, not again located in the middle. Therefore, it is *meant* only in normally distributed (symmetrical) data.

mean of a data set is usually denoted by $\overline{x}$, called " *x bar* ". The formula for calculating *mean* of single data (not grouped) is given by

$$\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

While *mean* for grouped data is given by the equation

$$\overline{x} = \frac{\sum_{i=1}^{n} f_i, m_i}{n}$$

f_i = frequency of each class

m_i = value of each class

Example 1

The Weight data of 9 male students in kilograms are 45, 39, 53, 45, 42 , 48, 51. 45, 40. Get *mean* from the student's weight data.

Solution:

Based on the *mean* that has been described in equation (1), the *mean* of the student's weight data is:

$$\bar{x} = \frac{the\ mean\ of\ weight\ students}{total\ all\ of\ the\ students}$$

$$= \frac{45 + 39 + 53 + 45 + 42 + 48 + 51 + 45 + 40}{9}$$

$$= 45.33$$

So, *the mean* weight of students is 45.33 kg.

Example 2

The following presents grouped data on the mathematics scores of grade VII students at SMP Abata. Get the mean from the following data.

	$\boldsymbol{f}$

)

Table 1. Data of Student Mathematics Values

Solution :

To be able to solve problems like this, we can use the formula in equation (2). But before that, to make the calculation process easier, we can add columns to table xx as shown in the following table.

ency	$\boldsymbol{f}$	**Point (m)**	$\boldsymbol{f, m}$

)

Frequency

Table 2. Data on Student Mathematics Values and their midpoint values

$$\overline{x} = \frac{\sum_{i=1}^{n} f_{i'} m_i}{n}$$

$$= \frac{133.5 + 272.5 + 387 + 670.5 + 676 + 665}{38}$$

$$= \frac{2804.5}{38}$$

$$= 73.80$$

So, the mean value of grouped data in table xx is 73.80.

B. Median The

Median is the middle value. The median divides the data into two equal parts. To determine the median, we must sort the data from smallest to largest, or vice versa. To get the median of a data, we must first pay attention to the number of data (n). In the case of single data, if n is odd, then the median is the datum that is right in the middle of the sorted data.

Data	Position

Table 3. Single data and information on the order/position of the data, with n

Pay attention to table 3, because n = 7, odd, then the median of the data is 18, or the fourth datum. In other words, if n is odd, then the median is $x_{\frac{n+1}{2}}$. Whereas For data with n , the median is calculated from the sum of the two datums in the middle of the sorted data, divided by 2. In other words, the median of data with n is $\left(x_{\frac{n}{2}} + x_{\frac{n}{2}+1}\right)/2$

Data	Position

Table 3. Single data and data sequence/position information, with n

Thus, in the example table 3, the median data is $\frac{18+15}{2} = 16.5$.

In the case of grouped data, special steps are needed to obtain the median. This is because

Step 1: Create a frequency distribution table with class and frequency intervals.

Step 2: Calculate the cumulative frequency of the data by adding the previous frequency value to the current value.

Step 3: Find the value of $n$by adding the values in frequency.

Step 4: Find the median class. If $n$is odd, the median is $\frac{n+1}{2}$. And if n is even, then the median is the average of the observations$n/2$ and $(n/2)$ + 1.

Step 5: Find the lower bound of the class interval and the cumulative frequency

Step 6: Apply the median formula for the grouped data, namely

$$Median = l + \left(\left(\frac{n}{2} - c\right)/f\right) \times h$$

With :

l = lower limit of the median class

n = number of data

c = cumulative frequency before class median interval

f = frequency of class interval

h = difference between the largest value and smallest value in the median interval class

Example:

Get the median from the following data on the Arabic language scores of grade IX students is:

Table 4. Data Value of Arabic Language

Value	Total of student

Solution:

To get the median for grouped data, we first need to calculate the cumulative frequency.

Value	Total of student	Frequency Kumulatif

)

In this case , , $n = 80$. To determine the position of the median class, we first get $n/2 = 40$. The 40th data position is in the third interval class. So from this information, we can write

$$l = 40$$

$$n = 80$$

$$c = 26$$

$$f = 37$$

$$h = 20$$

Based on the median formula in equation 3, we get

$$Median = l + \left(\left(\frac{n}{2} - c\right)/f\right) \times h$$

$$= 40 + \left(\left(\frac{80}{2} - 26\right)/37\right) \times 20$$

$$= 40 + ((40 - 26)/37) \times 20$$

$$= 40 + \left(\frac{14}{37} \times 20\right)$$

$$= 40 + 7.56$$

$$= 47.56$$

So, the median of the students' Arabic score data as shown in table 4 is 47.56 **C.**

C. Mode

The mode is also a measure of the value of data centering. This value gives us a rough idea of what datums in the data tend to appear most frequently. For example, a college offers 10 different courses to its students. Now, from this information, the courses that have the highest interest will be calculated as the mode of the data on the number of students taking each course. So overall, mode tells us about the highest frequency of any given item in the data set.

There are many real-life uses and the importance of using mode values. There are many aspects in which the mean or *mean* would not be suitable for describing a particular concentration of data. For example, take a look at the example given above. To find the highest number of students enrolled in a course, the mean or median will not represent it. Therefore, the mode value will be more suitable for this purpose.

Data can have more than one mode value, or have no mode at all. In summary, the data types based on the mode are as follows:

Unimodal data, namely data that has only one mode

- Bimodal data, namely data that has only two modes
- Trimodal data, namely data that has three modes
- Multimodal data, namely data that has more of three modes

To find a single data mode, we simply do this by sorting the data values from small to large or vice versa, then look for the value that occurs most often. The observation with the highest frequency is the mode value for the given data.

Example:

For example, given the shoe size data for class VII students as follows: 37, 40, 36, 38, 38, 39, 41, 35. Then the mode of the data is 38. Such data is called unimodal data, because the mode value is only one.

The mode for grouped data can be obtained using the following formula:

$$Modus = l + h\frac{(f_m - f_1)}{(f_m - f_1) + (f_m - f_2)} \quad (4)$$

atau

$$Modus = l + h\frac{(f_m - f_1)}{2f_m - (f_1 + f_2)} \quad (5)$$

dengan

l = batas bawah kelas interval yang mengandung nilai modus

h = ukuran kelas interval

f_m = frekuensi kelas interval yang mengandung nilai modus

f_1 = frekuensi kelas interval sebelum kelas interval modus

f_2 = frekuensi kelas interval setelah kelas interval modus

Example:

Look again at table 4. Get the mode from the grouped data

Solution :

Value	Total of student

)

From the information in the table, we can write

$l = 40$

$h = 20$

$f_m = 37$

$f_1 = 20$

$f_2 = 10$

$$Modus = l + h\frac{(f_m - f_1)}{2f_m - (f_1 + f_2)}$$

$$= 40 + 20\frac{(37 - 20)}{2(37) - (20 + 10)}$$

$$= 40 + 20\left(\frac{17}{74 - 30}\right)$$

$$= 40 + 20\left(\frac{17}{44}\right)$$

$$= 40 + 20(0.38)$$

$$= 40 + 7.72$$

$$= 47.72$$

The three measures of data concentration that have been presented in this chapter certainly have their own unique characteristics. These characters can then be associated with the type of data we are going to analyze. The following simple table shows the appropriate data center sizes for certain data types.

Tipe Data	Ukuran Pemusatan Data yang Cocok
Nominal	Mode
Ordinal	Median
Interval/Rasio (simetris)	*Mean*

Interval/Rasio (asimetris)	Median

EXERCISE

1.Determine the mean, median and mode of the following set of student answers to the question: "*How many times have you been late to school? this semester*?"

1, 1, 0.1, 2, 2, 0, 0, 0, 3, 3.0, 3, 3, 0.2, 2, 2, 1, 1,4, 1, 1.0,3, 0, 0, 0, 1, 1, 2, 2, 2, 2,1, 1, 1, 1, 4, 4, 4,1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2,1, 1, 1, 1, 1, 3,3,0, 3, 3, 1, 1, 1, 1.0, 0, 1, 1, 1, 1, 3, 3, 3, 2, 3, 3, 1, 1, 1,2, 2, 2,4, 5, 5, 4, 4, 1, 1, 1, 4,1, 1, 1,3, 3, 5, 3, 3, 3, 2,3, 3, 0, 0, 0, 0, 3, 3, 3, 3, 3, 3, 0, 2, 2, 2, 2, 1, 1, 1,3, 1, 0, 0, 0.1, 1, 3,1, 1, 1, 2, 2, 2, 4, 2, 2, 2, 1, 1, 1, 1.0, 0, 2, 2, 3, 3,2, 2, 3,2, 0, 0, 1, 1,3, 3, 3, 1, 1, 1, 1, 1,2, 2, 2, 2, 1, 1, 1, 1, 0.1, 1, 1, 3,1, 1, 1, 2, 2, 2, 1, 1, 1,2, 1, 1, 1,3, 3,5, 3, 3, 1, 1, 1, 3, 3, 3, 3, 1, 1, 1,4, 1, 1, 4, 4, 4, 4, 4, 4,1, 1, 1,2, 2,5, 5, 2, 3, 3, 4, 4,3,2, 2, 2, 1,5, 1,2, 2, 1, 1, 1, 2, 2, 2, 2, 2,1, 1, 0, 1, 1, 1,3, 3, 3, 3, 3

2.The following is data on the depth of dams in several regions in Indonesia

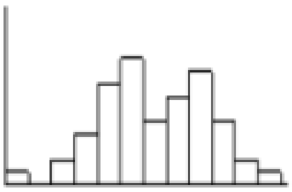
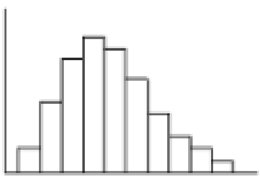
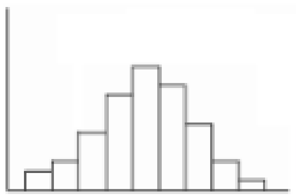
3.It is known that single data related to the mathematics scores of grade VIII students are as follows: 60, 70, 65, 50, 82, 91, 75, x, 100, 65. If the average value of the 10 students is 72, then determine value x.

4.Consider the following histogram. For each histogram, answer the following questions:

a. How would you describe the distribution of data on a histogram? Normal, Right skewed, left skewed, or bimodal?

b. Which data center measure is suitable to describe the distribution of the data?

c. Is the mean or median higher? Or is it the same value? Why? Tell.

BAB XIV
SOCIAL ARITHMATIC

This chapter discusses social arithmetic. Economic/trade activities are activities that are often carried out in daily life. We often carry out buying and selling activities so that we are familiar with the terms selling price, purchase price, profit, loss, discount and so on. These terms are part of the mathematics related to financial calculations in trade which is also known as social arithmetic. Before discussing it, it will be explained first what the meaning of p% is.

p: 100

Note: p is the amount of percentage

Example 1

Specify the following numbers in ordinary decimals!

a. 10% c. 35,25%

b. 9,5% d. 0,63%

Explanation:

a. 10% = 10: 100 = 0,1

b. 9,5% = 9,5: 100 = 0,095

c. 35,25% = 35,25: 100 = 0,3525

d. 0,63% = 0,63: 100 = 0,0063

Next, we will discuss how the decimal form is stated as a percentage. To convert decimal form to percentage form, you can use the following formula:

0% = A%

Note: A is the value in decimal

Example 2

Specify the following numbers in ordinary decimals!

a. 0,46 c. 9,4

b. 0,05 d. 0,7

Explanation:

a. 0,46 x 100% = 46%

b. 0,05 x 100% = 5%

c. 9,4 x 100% = 940%

d. 0,7 x 100% = 70%

In addition, it is necessary to remember again about arithmetic operations on integers, numbers, linear equations of one variable, and operations of algebraic forms.

A. Selling Price, Purchase Price, Profit and Loss

In buying and selling activities there are terms selling price/sales price, purchase price, profit and loss. For example, a seller buys an item for resale, if the selling price of the item is more than the purchase price, the seller earns a profit. On the other hand, if the selling price of the goods is less than the purchase price, the seller will suffer a loss. The following formula can be used to determine the purchase price, selling price and profit:

$$\boldsymbol{Profit = selling\ price - purchase\ price}$$

Example 3

A clothing seller named Mrs. Lara sells a shirt to a customer at a price of Rp. 92,500. The shirt he previously bought at a price of Rp. 75,000.00. How much profit did Mrs. Lara earn?

Explanation:

Selling price of clothes = 92,500

Purchase price of clothes = 75.000

Profit earned:

$$\text{Profit} = \text{selling price} - \text{purchase price}$$
$$= 92500 - 75000$$
$$= 17500$$

So that the profit earned by Mrs. Lara Rp17.500,00

Example 4

Rani is the owner of a shoe shop in town A, she sells a shoe for Rp. 63,000.00 and earns a profit of Rp. 22,500. Calculate how much capital he used to buy the shoes!

Explanation:

Selling price of shoes = 63,000

Profit = 22,500

Purchase price of shoes:

purchase price = selling price-profit

=63000-22500

=40500

So that the purchase price of the shoes is Rp. 40,500

Meanwhile, to calculate the loss, you can use the following formula:

Loss=purchase price-sale price

Example 5

Amanda bought a bag at an online store for Rp. 77.500.00, for some reason she resold the bag for Rp. 52.500.

Explanation:

The purchase price of the bag = 77,500

The selling price of the bag = 52,500

Make a loss:

Loss=purchase price-sale price

=77500-52500

=25000

So that Amanda suffered a loss of Rp. 25,000.00

Example 6

A toy shop owner sells toy cars for Rp. 35,000.00 and earns a loss of Rp. 2,500. Calculate how much capital he bought these cars!

Explanation:

Selling price of toy cars = 35,000

Loss = 2,500

The price of buying a toy car:

purchase price=sales price+loss

=35000+2500

=37500

So that the purchase price of a toy car is IDR 37,500

In addition to the selling price, purchase price, profit and loss, the terms profit percentage and loss percentage are also known. To calculate the percentage of profit can use the following formula:

$$\text{Profit Percentage} = \frac{\text{sales price} - \text{purchase price}}{\text{purchase price}} \text{x}100\%$$

Or

$$\text{Profit Percentage} = \frac{\text{Profit}}{\text{purchase price}} \text{x}100\%$$

Contoh 7

Look again at example 3. What percentage of profit will Mrs. Lara get if she buys clothes and then resells them at the purchase price and selling price as in example 3.

Solution:

Method 1:

Selling price of clothes = 92,500

Purchase price of clothes = 75.000

Percentage of profit earned:

$$\text{Profit Percentage} = \frac{\text{sales price} - \text{purchase price}}{\text{purchase price}} \text{x}100\%$$

$$= \frac{92500 - 75000}{75000} \text{x}100\%$$

$$= \frac{17500}{75000} \text{x}100\%$$

$$= 23{,}33\%$$

Method 2:

If the profit is known based on the calculation in example 3, it can be directly calculated as follows:

The profit obtained based on the calculation in example 3 Rp17,500, 00

Percentage of profit earned:

$$\text{Profit Percentage} = \frac{\text{Profit}}{\text{purchase price}} \text{x}100\%$$

$$= \frac{17500}{75000} \text{x}100\%$$

$$= 23{,}33\%$$

So that the percentage of profit earned by Mrs. Lara is 23.33%.

Example 8

Pak Arif bought shoes to resell. Mr. Arif's capital bought the shoes for Rp. 96,000.00 and earned a profit of 20%. Calculate the selling price of the shoes!

Explanation:

Purchase price of shoes = 96000

Profit:

Profit=Percentage of profit x purchase price

=20% x 96000

=19200

Selling price of shoes:

Selling price=purchase price+profit

=96000+19200

=115200

So the selling price of the shoes is Rp. 115.200.00.

Meanwhile, to calculate the percentage loss, you can use the following formula:

$$\text{Percentage Loss} = \frac{\text{purchase price} - \text{sale price}}{\text{purchase price}} \text{x} 100\%$$

Atau

$$\text{Percentage Loss} = \frac{\text{Loss}}{\text{purchase price}} \text{x} 100\%$$

Example 9

Look again at example 5. Calculate the percentage of losses suffered by Amanda in example 5!

Explanation:

Method 1:

The purchase price of the bag = 77,500

The selling price of the bag = 52,500

Percentage of loss obtained:

$$\text{Percentage Loss} = \frac{\text{purchase price} - \text{sale price}}{\text{purchase price}} \text{x} 100\%$$

$$= \frac{77500 - 52500}{77500} \text{x} 100\%$$

$$= \frac{25000}{77500} \text{x} 100\%$$

$$= 32{,}26\%$$

Method 2:

If the loss is known based on the calculation in Example 5, it can be directly calculated as follows:

The loss obtained by Amanda is based on the calculation in example 5 Rp.25,000.00

Percentage loss earned by Amanda:

$$\text{Percentage Loss} = \frac{\text{Loss}}{\text{purchase price}} \text{x} 100\%$$

$$= \frac{25000}{77500} \text{x} 100\%$$

$$= 32{,}26\%$$

So that the percentage of loss obtained by Amanda is 32.26%.

Example 10

Dinda bought clothes at a price of Rp. 75,000.00 to resell. What is the selling price of the clothes if it turns out that Dinda has suffered a loss of 10%?

Explanation:

Buying price of clothes = 55.000

Make a loss:

Loss=Percentage of loss x purchase price

$$= \frac{10}{100} \text{ x } 75000$$

$$= 7500$$

Selling price for clothes

Selling price=purchase price-loss

=75000-7500

=67500

So the selling price of the clothes is Rp. 60,500.

A. Rebate (Discount), Gross, Net and Tara

Another term that we often hear in the economy is Rebates/Discounts. Rebates / discounts are discounts on sales prices at the time of buying and selling transactions. The difference between a rebate and a discount lies in the number of items that are being traded. Discount is a discount term for an item, while rebates are usually for discounts on items that are more than one or wholesale items. Almost all sales sectors apply these rebates/discounts in order to attract buyers' interest, including selling household appliances, clothing, food and beverages, electronics, services, and others. To determine the rebate/discount, you can use the following formula:

discount =Price before discount-price after discount

Example 11

A shoe has an initial price of Rp. 85,000.00. To attract buyers, the seller lowers the price to Rp. 68,000.00. Calculate how much discount the seller gave for the shoes?

Explanation:

Price before discount = 85,000

Price after discount = 68.000

Discount for the shoes:

Discount = price before discount-price after discount

=85000-68000

=17000

So the shoes are subject to a discount of Rp. 17,000.00.

The clothing store "Jaya Abadi" is holding a massive promo on the various items it sells. One batik suit for brand X, which was originally priced at Rp. 370,000.00, is given a 20% discount. If Mira wants to buy the item. how much does Mira have to pay?

Explanation:

Price of batik suit before discount = 370,000

Discount = 20% x 370,000 = 74,000

Price after discount = price before discounts

=370000-74000

=296000

So the price to be paid by Mira is IDR 296,000.00.

In addition, related to the weight of an item there are terms Gross, Net and Tara. Gross is defined as gross weight, namely the weight of an object and its packaging, net or net weight, namely the actual weight of an item without its packaging, and tara is the difference between gross and net or can be interpreted also with the weight of the packaging alone. The formula that can be used to determine gross, net and tare is as follows:

Net = Gross-Tara

Example 13

An item is known to have a gross of 50 kg, the equivalent of 1.5% of its gross. How much is the net?

Explanation:

Gross = 50 kg

Tara = 1.5% x 50 kg = 0.75 kg

Net = Gross-Tara

=50-0.75

=49.25

So the net is 49.25 kg

Example 14

An item has a gross of 7.5 kg, net 6.9 kg. How much is the tare?

Explanation:

Gross = 7.5 kg

Net = 6.9 kg

$$\text{Tara} = \text{Gross} - \text{Net} = 7.5 - 6.9 = 0.6$$

$$= 7.5 - 6.9 = 0.6$$

$$= 0.6\text{So the Tara is } 0.6 \text{ kg}$$

$$\text{So the Tara is } 0.6 \text{ kg}$$

Example 15

One sack of packaged rice is known to have a net of 25 kg and the equivalent is 1.2% of the net. Calculate the gross of one sack of rice!

Explanation:

Net = 25 kg

Tara = 1.2% x 25 kg = 0.3 kg

Gross =Net+Tara

=25+0.3

=25.3

Up to Gross25.3 kg

B. SINGLE INTEREST

In everyday life, it is not uncommon for us to make transactions at a bank. In these transactions, a person can save or borrow money from a bank and a bank usually applies interest. Interest is the money that the borrower pays to the owner of the money other than the principal. If we save money in the bank, our money will increase because we get interest. In addition, if we are going to borrow money from the bank, interest will be charged.

The type of interest on savings and loan interest to be obtained can be in the form of single interest or compound interest. In this book, we will only discuss single interest or interest on capital. Single interest is the interest that arises at the end of a certain period of time which does not affect the amount of capital borrowed. per year, and the amount of interest is expressed by I, then:

$$I = M \ x \ P\%$$

Note: The interest percentage is always stated for one year unless otherwise stated in the problem.

Example 16

Anisa has a savings account at Bank “Sejahtera” amounting to Rp. 500,000.00 with an interest rate of 10% per year. Calculate the amount of money Anisa has after 6 months.

Explanation :

1 year interest:

$$I = M \ x \ P\%$$

=500000 x 10%

=50000

6 months interest = 6/12 x 50000

= 25000

So that the amount of Anisa's money after being kept for 6 months at the "Sejahtera" Bank becomes:

Rp500.000,00 + Rp25.000,00 = Rp525.000,00

Single interest consists of several types, namely ordinary interest, exact interest, simple interest, semiannual interest and annual interest. The following describes the various types of flowers.

1. Annual interest is interest based on calculations in a year with the formula as written above.
2. Ordinary interest is the interest calculated based on the commercial year. After a day, the amount of interest can be calculated as follows:

$$I = \frac{s}{360} x\ M\ x\ P\%$$

Note: if one year there are 360 days.

Example 17

Andika saves money in a bank for Rp. 1,500,000.00 with an interest rate of 6% per year. How much money Andika after saving 2 months?

Explanation:

Money saved = 1500000

Saving time = 2 months = 60 days.

The amount of interest after 60 days is

$$I = \frac{s}{360} x\ M\ x\ P\%$$

$$= \frac{60}{360} x\ 1500000\ x\ 6\%$$

$$= 15000$$

So Fani's amount of money after 2 months becomes:

Rp.1,500,000.00 + Rp.15.000,00 = Rp.1,515,000.00.

3. Exact interest is interest calculated based on the exact number of days in a year, i.e. 365 days, except for leap years of 366 days.

After s days, the amount of interest:

$$I = \frac{s}{365} x\ M\ x\ P\%$$

Note: if one year is 365 days (column year)

Or

$$I = \frac{s}{366} x\ M\ x\ P\%$$

Note: if a year has 366 days (leap year)

Simple interest is interest at the end of a certain period which is calculated from the principal, the interest is equal to the product of time, interest rate and principal.

Example 18

Adhar saves money in a people's bank for Rp. 3,500,000.00 with an interest rate of 5% per year. How much interest did Adhar save for 4 years?

Explanation:

Adhar interest for a year:

$$I = M\ x\ P\%$$

$$= 3500000\ x\ 5\%$$

$$= 175000$$

So the interest on Adhar money for 4 years = 4 x 175000 = IDR 700,000.00

4. Semi-annual interest is the interest rate charged and calculated every six months.

After t months, the amount of interest:

$$I = \frac{t}{12}\ x\ M\ x\ P\%$$

Example 19

Mrs. Nina wants to borrow money from the bank for Rp. 11,000,000.00 with a loan period of 6 months. The bank gives an interest of 25% per year. How much money must Mrs. Nina return to the bank?

Explanation:

The amount of interest that must be paid by Mrs. Nina to the bank:

$$I = \frac{t}{12}\ x\ M\ x\ P\%$$

$$= \frac{6}{12}\ x\ 11000000\ x\ 25\%$$

$$= 1375000$$

The amount of money that Mrs. Nina returned to the bank after 6 months became:

Rp11.000.000,00 + Rp1.375,00 = Rp12.375.000,00

PRACTICE

1. Mrs. Lita bought clothes for Rp. 52,500.00 to resell. When selling it to a buyer, the clothes sell for Rp. 67,000.00. Did Mrs. Lita experience gain or loss? How much profit/loss did Mrs. Lita get?
2. Raka buys watches to resell. The purchase price of the watch was Rp. 230,000.00 and Raka made a profit of 18%. What is the selling price of the watch?
3. A fruit seller sells pears at a price of Rp. 27,000.00/kg. How much capital did he buy 1 kg of pears if from the sale he suffered a loss of IDR 8,500?
4. Bara buys goods whose initial price is Rp. 105,000.00. After getting the discount he only paid Rp. 84,000.00. What percent discount does Bara get?
5. Nina buys goods at a shop, with a 25% discount she only pays Rp. 240,000.00. What was the initial price of the item?
6. Ijul bought 3 sacks of rice. One sack has a net of 25.5 kg and a gross of 25.8 kg. How much is the tare from the 3 sacks of rice?
7. Arif was asked by his mother to buy 2 sacks of flour for his family's traditional snack business. Each sack of flour has a net weight of 50 kg. What is the gross of the 2 sacks of flour if the tare of each sack is 1% of the net?
8. A sack of rice has a gross weight of 10 kg and the equivalent is 0.5% of the gross. How much is the net?
9. Mrs. Ani borrowed from a bank as much as Rp. 2,400,000.00 with an interest of 25.5% per year. BuAni must repay the loan and interest in monthly installments for 1 year. How much is Mrs. Ani's installment?
10. Laras keeps money in a bank as much as Rp. 550,000.00, it is known that the bank's interest is 20% per year. Without calculating the interest for 1 year, calculate the interest on Laras money after 4 months?

Bibliography

Adjie, Nahrowi dan R. Deti Rostika. (2009). *Konsep Dasar Matematika*. Bandung: UPI Press

De Walle, John A. Van. (1990). *Elementary School Mathematics*. White Plains, New York

Musser, G. L., & Burger, W. F. (1991). *Mathematics for elementary teachers*. Neew York: Maxwell Macmellan International.

Musser, G. L., Burger, W. F., & Peterson, B. E. (1988). *Mathematics for elementary teachers:* A contemporary approach. New York: Macmillan.

Polya, G. (2014). *How to solve it: A new aspect of mathematical method*. Princeton university press.

Soedjadi, R. (2000). *Kiat Pendidikan Matematika di Indonesia. Jakarta*: Direktorat Jenderal Pendidikan Tinggi Departemen Pendidikan Nasional.

Soedjadi, R. (2007). *Masalah Kontekstual Sebagai Batu Sendi Matematika Sekolah. Depdiknas*, Unesa, dan PSMS.

Sibarani, Maslen. 2014. *Aljabar Linier*. Penerbit Raja Grafindo Persada, Jakarta.

Sutojo, dan Bowo. 2010. *Teori Dan Aplikasi Aljabar Linier Dan Matriks*. Penerbit : Andi Publisher

Novia Ariyanti, Nuril Lutvi Azizah, *Buku Ajar Mata Kuliah Matematika Diskrit* , Umsida Press: 2020

Adji, Nahrowi. *Pemecahan Masalah Matematika Bandung*: UPI Press, 2008.

Ina V.S. Mullis. Et al. *TIMMS 2011 International Result in Mathematics.* Chestnut Hill, MA: Boston College., 2012.

NCTM. *The Open-Ended Approach: A New Proposal for Teaching Mathematics*. ed. Becker and Shimada.

Reston: NTCM INC.

Polya, G. *How to Solve It: A New Aspects of Mathematical Method*. New Jersey: Princeton University Press, 1985.

Sudiarta, Igusti Putu. *Pengembangan Model Pembelajaran Matematika Berorientasi Pemecahan Masalah.*

Kontekstual Open Ended. *Jurnal Pendidikan dan Pengajaran*, Vol. 38, no 1. Tahun 2005, h, 582.

Suherman, Erman. *Strategi Pembelajaran Matematika Kontemporer*. Bandung: UPI, 2003.

GLOSARIUM

Astronomy : The study of everything in the universe

Natural Number : The number used to count or order

Integers : a number with no decimal or fractional part and it includes negative and positive numbers.

Whole Number : The numbers that include natural numbers and zero

Rational Number : A type of real number, which is in the form of p/q where q is not equal to zero.

Real Number : The number that can be written in decimal

Factorisation : A method of breaking the arithmetic algebraic expressions into the product of their factors.

Geography : The study of the physical features of the earth and its atmosphere, and of human activity as it affects and is affected by these, including the distribution of populations and resources, land use, and industries.

Fraction : The representation of the parts of a whole or collection of objects.

Gupta Period : The glory era of Indian mathematicians in the Gupta period made important contributions.

Relation : It relates to the relationship between sets of values of ordered pairs.

Inverted relation : The inversion of measured relation.

substitution : a way to solve a linear system algebraically. The substitution method works by substituting one y-value with the other.

trigonometry : The branch of mathematics concerned with specific functions of angles and their application to calculations.

Learning Materials Development

Three Languages: Indonesia, Arab, & English

MATHEMATICS

for

Madrasah Tsanawiyah (MTs) & Aliyah (MA)

Universitas Islam Negeri (UIN) Mataram
UPT. Pengembangan Bahasa (UPB)
Pendidikan Bahasa Arab (PBA)
Tadris Matematika (TMTK)
Tadris Bahasa Inggris (TBI)
Jalan Gajah Mada No.100 Jempong Baru-Mataram
Website: www.uinmataram.ac.id

University of Science & Technology, Yemen (USTY)

Learning Materials
Development Three Languages
(Indonesia, Arab and English)

MATHEMATICS

for Madrasah Tsanawiyah (MTs) & Aliyah (MA)

ISBN 978-623-317-332-2

9 786233 173322