

A framework for Assessing Translation among Multiple Representations

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ABSTRACT

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Translation among multiple representations is one of the important abilities to be possessed by prospective mathematics teachers. The mathematics learning curriculum ranging from elementary to college level needs to emphasize the ability to translate mathematical ideas in a form of representation to different mathematical representation structures. Assessing of this ability is important because it can be used as a basis for developing other mathematical literacy skills. The purpose of this study is to describe the level of translation between mathematical representations of prospective mathematics teachers. A descriptive qualitative study was conducted to forty prospective mathematics teachers in State Islamic University of Mataram. The selection of research subjects was carried out in a purposive sampling. Data were obtained from mathematical translational thinking task in the form of an assignment sheet consisting of 2 questions related to quadratic equations and task-based interviews. Data were analyzed using fixed comparison analysis, which is comparing incidents (phenomena) with the aim of classifying data. The study found five levels of translational ability between mathematical representations, which were named: level 0, level 1, level 2, level 3 and level 4. The characteristics of each level are built based on four mathematical translational thinking processes: unpacking the source of representations, coordinating initial stage, design the target representation and determine the equivalence between the source and target representations. The findings in this study could be used as a guide to develop an instrument to assess mathematical translation ability.



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A. INTRODUCTION

Research at the end of this decade has studied a lot about mathematics content knowledge, which is a basic competency that prospective mathematics teachers must possess. This knowledge is the basis that a person must have to be called mathematically literate (Reid & Reid, 2017). Mathematical content knowledge is a significant aspect for prospective mathematics teachers in understanding learning situations and planning educational actions to encourage learning (Dunekacke et al., 2015). Thus, knowledge of mathematics content is important to be a concern in the teacher education curriculum.

Knowledge of mathematical content covers various aspects, including: the ability to describe mathematical relations, communicate mathematical knowledge and operate

mathematical objects. This ability can be achieved by learning that emphasizes the use of various types of mathematical representations in solving problems. According to Rahardi et al., (2018) there are seven ways to represent mathematical ideas, namely: (a) real world situations, (b) manipulative models, (c) pictures or diagrams, (d) writing special symbols, (e) inductive, (f) mathematical symbols and procedures, and (g) spoken words. The development of students' ability to move between and within these representations can improve their mathematical understanding both conceptually and procedurally.

Students' proficiency in using mathematical concepts and procedures in solving problems depends on their fluency in translating problems into mathematical models (Bal, 2015). Mathematical problem modeling is related to the activity of constructing different mathematical representations. For example, create a symbolic model (symbolic representation) of real world situation problems (verbal representation). The cognitive process in changing one form of mathematical representation to another from a different mathematical representation is called translation between mathematical representations (Bossé et al., 2014). The term used in this study is mathematical translational thinking.

Some researchers define mathematical translational thinking. According to Wibawa et al., (2020) mathematical translation is a psychological, intellectual or cognitive process associated with changing the information encoded in one mathematical representation (source) to another mathematical representation (target). Another definition stated by Bossé et al., (2014) which views mathematical translation as a process in which constructions or ideas formed from a mathematical representation (source) that have been successfully reformulated into a targeted mathematical representation, using structures and characteristics that are in accordance with the targeted representation, it means that mathematical translation activities involve at least two different types of representations. The translation between the two forms of representation involves several cognitive activities, such as mapping or constructing ideas from one form of representation to another (Adu-Gyamfi et al., 2012).

Furthermore, Adu-Gyamfi et al., (2019) argues that translation between mathematical representations is a process by which concepts and attributes that are meaningful in (initial) mathematical representations are interpreted or mapped into correlated structures and attributes in other (final) representations through the use of appropriate heuristics. The initial representation is called the source representation and the final representation is called the target representation.

Several studies have shown that mathematical translational thinking can improve mathematical connection and problem solving abilities and develop deep conceptual understanding (Adu-Gyamfi et al., 2017; Bal, 2015; Jao, 2013). Beside that, the mathematics learning curriculum from elementary school to university level needs to emphasize the ability to translate mathematical ideas in a form of representation to a different mathematical representation structure (Adu-Gyamfi et al., 2012). This suggests that it is very important to train mathematical translational thinking to be developed in learning mathematics, even though the mathematics curriculum in Indonesia has not been explicitly mentioned. However, it can be implicitly integrated into basic competencies or learning outcomes in several courses.

Thus, mathematical translational thinking is one of the abilities that prospective mathematics teachers must possess.

The study of mathematical translational thinking is still rare in Indonesia, although it has been widely discussed abroad. In the Indonesian context, research by Sa'Dijah et al., (2018) and Afriyani et al., (2019) has found several characteristics of students' mathematical translational thinking, but studies on how to measure the quality of thinking skills have not been explored much. Therefore, further studies are needed on the framework of mathematical translational thinking levels that can be used as a basis for assessing mathematical translational thinking skills. The development of this grading framework can be used to see the degree of achievement of students' mathematical translational thinking processes, and can be used as a barometer to find students' strengths and weaknesses in mathematical translation.

The framework can be used as an instrument to determine the level of response or processes of mathematical translational thinking. The framework used in this study is the SOLO taxonomy (Structure of Observed Learning Outcomes) developed by Briggs and Collis (1982). This taxonomy was chosen because it can be used to determine measurable levels of thinking skills. The SOLO taxonomy consists of five levels, namely pre-structural, uni-structural, multi-structural, relational, and extended abstract. The characteristics of each level are determined based on the mathematical translation mechanism according to Bossé et al., (2014) which includes the following stages: (1) unpacking the source representation, (2) carrying out initial coordination, (3) building the target representation, and (4) determine the equivalence between the source and target representations. The grading framework that will be developed in this research consists of levels of mathematical translational thinking along with their respective characteristics.

B. METHODS

This research is a grounded theory research through a qualitative research approach. The research design used is Systematic Design. According to (Creswell, 2012), a systematic design in Grounded Theory emphasizes the use of data analysis stages in open, axial and selective coding and uses the development of paradigms or logical visual images from generalized theories.

This study was 40 prospective mathematics teachers from the State Islamic University of Mataram. The selection of participants was carried out by purposive sampling, namely taking research subjects based on certain considerations where if the results of the categorization did not meet a minimum of two people at each level, then the subject was selected again until each level could be represented by a minimum of 2 students. The research subjects were 2 students at each level who had almost the same pattern of answers and reasoning. Data were collected using a test containing a mathematical translational task involving graphic and symbolic representations and a task-based interview.

Initial research was conducted by reviewing the theory of mathematical translational thinking as the basis for drafting a framework for ranking mathematical translational thinking skills. The process of analyzing the validity and reliability of the grading framework is carried out using the constant comparative method. This method consists of four stages: "(1)

comparing the incidences that apply to each category, (2) integrating the categories and their properties, (3) limiting the theory, and (4) writing the theory. Throughout the four stages of the constant comparative method, the researcher continuously sorts through data collection, analyzes and encodes information, and reinforces theory generation through a theoretical sampling process (Kolb, 2012).

Validation of the theoretical draft of this framework was carried out to test the content and construct validity of the draft theory. Validation involves 1 expert in leveling theory and 1 expert in translational thinking theory between mathematical representations. The two validators gave an assessment stating that the draft theory of grading qualifies as a logical and theoretically rational thought. All notes from this validator are then used as the basis for revising the draft leveling theory.

The instrument used is a mathematical translational thinking task in the form of an assignment sheet consisting of 2 questions related to quadratic equations. There are two types of questions given, namely translation from symbolic to graphic and translation from graphic to symbolic. This instrument was validated by 1 mathematician. There are two aspects that are assessed deeply by the validator on this instrument, namely the construction aspect and the language aspect. The results of the validator's assessment indicate that this instrument is feasible to use with improvements. This instrument was then revised according to the validator's input before taking data.

C. RESULT AND DISCUSSION

The initial theory that was used as a reference in developing a grading framework for mathematical translational thinking skills in prospective mathematics teacher students was the mathematical translation process (Bossé et al., 2014) and the SOLO taxonomy. Based on these two theories, a hypothetical framework for grading mathematical translational thinking skills was developed for prospective mathematics teacher students, as shown in Table 1.

Table 1. Mathematical Translation Thinking Skills Framework

Level of Mathematical Translational Thinking Ability	Indicator
Level 0	<ul style="list-style-type: none"> – Able to identify information on the task, but unable to identify micro-concepts in source representation. – Unable to devise a strategy to build an appropriate target representation.
Level 1	<ul style="list-style-type: none"> – Able to identify information on the task. – Able to identify one micro concept in source representation. – Able to design simple strategies in building target representation, but unable to predict micro concept of target representation.
Level 2	<ul style="list-style-type: none"> – Able to identify information on the task. – Able to identify several micro concepts in source representation. – Able to determine the interrelationships between micro-concepts in the source representation. – Able to design strategies to build target representations and be able to predict micro-concept target representations. – Haven't been able to create a network of ideas between source and target representations.

Level of Mathematical Translational Thinking Ability	Indicator
Level 3	<ul style="list-style-type: none"> – Able to make target representation, but still not precise. – Able to identify information on the task and can identify various micro concepts in source representation. – Able to determine the interrelationships between micro concepts in the sources representation appropriately. – Able to design strategies to build target representation and be able to predict the micro concept of target representation. – Able to create a network of ideas between source and target representations. – Able to build target representation appropriately.
Level 4	<ul style="list-style-type: none"> – Able to identify information on the task and can identify various micro concepts in source representation. – Able to determine the interrelationships between micro concepts and micro concepts in the sources representation appropriately. – Able to design strategies to build target representations and be able to predict micro-concept of target representations. – Able to create a network of ideas between source and target representations. – Able to build target representation appropriately. – Able to check the suitability of mathematical ideas contained between the target representation and the source representation. – Able to create flexible connections between mathematical representations and be able to generalize to various mathematical concepts.

The developed mathematical translational thinking ability framework consists of five levels of ability, starting from level 0 to level 4. The mathematical translational thinking ability level 0 is a form of response to the previous mode at the pre-structural level in SOLO Taxonomy. While the ability to think mathematically translational level 1, 2 and 3 is a response to the target mode in SOLO Taxonomy, namely Unistructural, Multistructural and Relational. The level in this target mode becomes the response that will be addressed in learning. Finally, there is the ability to think mathematically at level 5 which is the next mode at the Extended Abstract level in SOLO Taxonomy. This last level becomes a high-level response that may be achieved by students. Indicators at five levels are of course related to the four stages of the mathematical translation process consisting of: dismantling sources, performing preliminary coordination, establishing targets, and determining equality.

The test was given to forty prospective mathematics teachers to determine the existence of the level of mathematical translation thinking skills. All subjects were classified based on their answers adjusted by leveling indicators. The results of the classification of mathematical translational thinking skills in all research subjects are described in Table 2.

Table 2. Mathematical translation task results

No	Level	n	Percentage
1	Level 0	7	18%
2	Level 1	12	30%
3	Level 2	14	35%
4	Level 3	5	13%
5	Level 4	2	5%
Total		40	

Based on Table 2, it is known that each level meets a minimum of 2 students and most students are at level 2. Of the 40 students who were used as research subjects, 2 people from each level or as many as 10 students were analyzed further. The responses for each level of mathematical translational thinking are described in Table 3.

Table 3. Subject responses for each level of mathematical translational thinking skills

Level	Response of each subject
Level 0	<p><u>S1 response:</u> S1 only quotes and processes the information contained in the questions using irrelevant strategies. S1 does not seem to understand the concept of quadratic equations correctly.</p> <p><u>Response S2</u> The argument given by S2 is still very simple. In answering the problem of symbolic to graphic translation, S2 was not able to correctly identify the micro concept of the source. Whereas in the graphic to symbolic translation, S2 was able to identify one micro concept of the source but could not proceed to the initial coordination process.</p>
Level 1	<p><u>S3 Response</u> S2 is only able to identify one micro concept of source representation, namely determining the factoring form of the quadratic equation. S2 tries to create a target representation, but it still goes wrong.</p> <p><u>S4 Response</u> S4 is able to identify one micro concept correctly, namely the roots of a quadratic equation, but it cannot be used to construct a graph (target representation) correctly.</p>
Level 2	<p><u>S5 Response</u> S5 unpacks the symbolic source representation by identifying the form of a quadratic equation. Furthermore, S5 determines the form of factors and solutions to the roots of the quadratic equation correctly. Then, S5 relates the root solution to the point where the curve intersects the x-axis. The strategy design to build the target (symbolic representation) is done by identifying the x- and y-intercept points and determining the peak point. It's just that S5 is wrong in calculating the y-intercept and the y-value for the vertex of the curve. S5 also fails to render correct target representation</p>

Level	Response of each subject
	<p><u>S6 Response</u> S6 is able to correctly identify the micro concept of source representation. It's just that the S6 looks still confused in linking the source and target representation ideas. S6 knows about the concept of determinant D and coefficient a in quadratic equations, it's just that they have not been understood correctly in terms of constructing the graph correctly</p>
Level 3	<p><u>S7 Response</u> S7 is able to correctly identify the micro concept of source representation. S7 can determine the roots of the quadratic equation and perform initial coordination by looking at the value of the coefficient a in the quadratic equation. This is done to find out whether the curve will open up or down. In designing the target representation, S7 calculates the intersection point with respect to the x and y axes as well as the vertex of the curve. S7 then graphs the quadratic function based on this information.</p> <p><u>S8 Response</u> When identifying the micro concept of source representation, S8 makes a factoring form of the quadratic equation so that the roots of the solution are obtained. Based on this information, S8 designs the target representation by determining the position of the roots on the graph and calculating the vertex of the graph. The S8 can create a properly scaled target representation.</p>
Level 4	<p><u>S9 Response</u> S9 can immediately identify the source representation by looking for the roots of the given quadratic equation. Next, S9 designs the target representation by calculating the vertex of the function. The S9 then creates an exact representation of the target. S9 can relate the idea of the coefficient a in the quadratic equation and the value of the determinant of D with possible graphic forms. This is used as the basis for looking at the equivalence between the source and target representations. The S9 can develop ideas about how to quickly translate from symbolic to graphic and vice versa.</p> <p><u>S10 Response</u> S10 is able to design a graph of a function of a quadratic equation by directly considering the value of the coefficient a and the value of the determinant D. In addition, S10 also determines the roots of the quadratic equation and the vertex of the function. From this information, the S10 can make an accurate representation of the target. Furthermore, S10 confirms the suitability of the answer with the question, meaning that S10 is able to determine the equivalence between the translated representations.</p>

In general, the stages in the mathematical translational thinking process consist of: unpacking the source representation, initial coordination, building the target representation and determining the equality between the source and target representations. Based on the responses given by students (research subjects) in Table 3 it appears that the simplest stages

in mathematical translational thinking can be done by students at Level 0, and the complete stages can be done by students at Level 4.

At the stage of dismantling the source representation, students identify micro-concepts in the source representation. This stage is carried out by students at all levels, except level 0. Students identify the micro concept of source representation by finding out what important information is contained in the source representation. For example, identifying the roots of a quadratic equation based on the intersection points on the x-axis on the graph.

The initial coordination stage is carried out by students starting from level 2. Several micro concepts in the source representation are used to design the target representation. This stage is carried out simultaneously by predicting the micro concept of the target representation which is part of the stage of building the target representation. At this stage, the strategies used by students at each level are somewhat different. At level 2, students calculate the point of intersection of the x and y axes using a formula. While at levels 3 and 4, students can directly predict the shape of the graph by looking at the form of the equation and calculating the value of the square roots. This is in accordance with the opinion of Bossé et al., (2014) which states that students' thinking processes in each step of translation from graphic to symbolic vary. For example, at the initial coordination stage, students in the low and middle groups chose to do fact mapping, while students in the high group did concept mapping.

The design of the target representation is done completely by students at levels 3 and 4. Students at levels 0 to level 2 have not correctly constructed the target representation, because at some stage in the design of the target representation, students encounter errors in some procedures. For example, in translating a graph into a symbol, students fail to determine the exact quadratic equation of the available chart due to their inability to relate the concept of coefficient a in the quadratic equation with the shape of the graph. At level 3 and 4 students can build target representations correctly because they can determine the equivalence of the two representations.

The last stage of the mathematical translational thinking process is to determine the equivalence of the source and target representations. This stage is correctly carried out by students at level 4. Determining the equivalence between the source and target representations is done by ensuring that what is produced in the target representation meets what is requested in the source representation. In the translation from symbolic representations to graphs as in the questions given, students are able to ensure that the graphs made are in accordance with their symbolic forms in the form of quadratic functions as given in the problem. Students can find that the shape of the two parabolic curves intersect each other which is known by looking at the common points on the two functions. In addition, students at level 4 can develop their ideas about an easy way to graph a function by identifying several points that may be found in a function. Students can also estimate the shape of the graph from the value of the coefficient a in front of x^2 , it means that students are able to create flexible connections between two different mathematical representations. In accordance with the opinion of Bossé et al. (2014) which states that in performing mathematical translations, one must not only know or recognize the concepts contained in the source representation by identifying and coding them, but also be able to relate these

concepts into different forms according to the intended representation or target representation.

According to the SOLO taxonomy, the response that is expected to appear in learning is called the target mode. This response can be seen at level 1, level 2 and level 3 in the framework for grading mathematical translational thinking skills. At these levels, the ability to think mathematically translationally develops in stages in the four stages that characterize it. Finally, there is the next mode, namely at level 4 thinking skills which is the application of the extended-abstract level in the SOLO taxonomy. This highest level is the embodiment of complete mathematical translational thinking skills, where students are able to develop their ideas more broadly beyond what is expected.

The existence of tiers in the ability to think translationally is basically needed in the assessment process, especially in making assessment rubrics. Educators (teachers or lecturers) can develop learning designs based on the results of an assessment of the abilities of their students. Conceptual and factual knowledge should be presented in a balanced way. As Bromley (2015) argues, it is necessary to teach less subject matter that is discussed in more depth and provides many examples where these concepts work so as to provide a solid basis for the development of factual knowledge. Furthermore, this leveling can also help students understand new concepts or information, develop a deep foundation of factual knowledge, understand facts and ideas in the context of a conceptual framework, organize knowledge and develop meta-cognition. So this certainly takes considerable control over the setting of learning objectives and monitoring the progress of their learning outcomes. This tiering theory can be used as a reference for teachers/lecturers in designing appropriate lecture designs. If teachers know about potential barriers to understanding before lessons are taught, they can develop well-developed lesson plans and use modified teaching strategies to help students overcome or at least minimize these barriers (Ahmed et al., 2020).

This research is still possible to be developed further because the discussion is still limited to the translation from symbolic representations to graphics. So that future research can develop broad ideas, for example related to developing assessments that refer to the tiering framework developed in this study. In addition, it can also be expanded the idea of study on instructional design and media that facilitate students according to their level of thinking ability.

D. CONCLUSION AND SUGGESTIONS

The results of this study found five levels of mathematical translational thinking skills, which were named: level 0, level 1, level 2, level 3 and level 4. The determination of indicators for each level is based on four stages in mathematical translational thinking, namely: unpacking source representation, perform initial coordination, design the target representation and determine the equivalence between the source representation and the target representation. Based on the framework that has been found in this research, further studies can be carried out, especially on the development of mathematical translational thinking ability instruments. Research on various mistakes made by subjects at low levels also needs to be explored deeply.

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