Sensorimotor-Based Digital Media:
An Alternative Design of Digital Tools in Mathematics Education

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Abstract—**Regarding the controversy on the benefit of digital technology in mathematics education, the way (what and how) a digital tool presented to students is the primary matter determining the effectiveness of such a tool. This paper suggests that sensorimotor based digital tools supporting comprehensive experiences to their users (i.e. perceptual, sensory and motoric experiences) potentially foster students' cognitive development. Such tools are appropriate for learning abstract and mental entities, such as mathematics, for their power to embody abstract concepts.**

Keywords: instrumental approach, embodied approach, instrumental genesis, embodied cognition

I. INTRODUCTION

The benefit of using digital technology is subject to controversy. Reference [1] reports that the use of computer in education does not significantly improve students' achievement. On the other hand, [2] and [3] claim that digital tools effectively foster students' achievement. It is argued that the way technology is presented and employed to students is the factor determining the effectiveness of such a technology in education [4].

In the context of integrating digital technology in the mathematics classrooms, [5] highlights that the technology ought not to be used as a replacement for basic mathematical understandings and intuitions; instead, it should be used to foster such understandings and intuitions. However, how to develop and orchestrate such digital tools to promote students' mathematical intuition and conceptual understanding is blurred and less studied yet. As it can alter mathematical activities [6], the vast availability of digital mathematical tools influences mathematics education. Nevertheless, how to exploit the potential of these powerful tools for mathematics learning is much unknown and becomes a big challenge of researchers in the field [7]. Therefore, the question of how to integrate digital technologies to foster mathematics learning is waiting to be answered.

In line with this challenge, [8] promotes the didactical functionality of digital technologies in mathematics education. It asserts two classifications

of the functionality of digital technology in mathematics education, such as digital technology for doing mathematics and for learning mathematics.

The digital technologies for doing mathematics refers to the digital tools that use to help students to perform mathematical tasks, such as counting and graphing. Here, students and the tools share functionality in the learning, where the tools are treated as the assistance for students once they deal with mathematical problems. For instance, once the focus of the learning is a mathematical problem solving, a digital tool, such as a digital calculator or another computer software, is used to help students to perform complex computations while the students think of the strategies to solve the problem. In this case, the tool does not target the core of the mathematical activity itself (i.e. the problem solving), but concerns as the outsourcing part of the work to relieve the student's mind.

Meanwhile, digital technologies for learning mathematics refers to the didactical use of digital tools for learning mathematical concepts. This functionality is classified into two types, such as tools for practising mathematical skills (skill-oriented tools) and tools for developing students' conceptual understanding of mathematics (concept development tools). The digital tools for practising mathematical skills focus on enhancing students' capability of particular mathematical skills. Here, the tools are used as personal learning environments that allow students to experience varied mathematical tasks, provided feedbacks that help students to learn from mistakes.

Meantime, the digital tools for promoting students' conceptual understanding of mathematics refers to the use of digital tools to support students acquiring conceptual understanding of a particular mathematical concept (concept development). It involves using a digital tool to explore phenomena that invite conceptual development. Such use of digital tools is in line with [5] that the use of digital tools should promote students' mathematical conceptual understanding and intuitions.

Moreover, the use of digital tools for fostering students' conceptual understanding is the primary

purpose of integrating digital technologies in mathematics classrooms [5] [7] [8]. However, this didactical function is considered as the most challenging to exploit since concept development is regarded as a higher-order learning goal [7]. Therefore, the current paper aims to elaborate theoretical based arguments to address the challenge. The main question being investigated is the answer to the following question: What are the characteristics of a digital learning tool to foster students' conceptual understanding of mathematics?

II. ISSUES OF INTEGRATING DIGITAL TOOLS

Reference [1] reports that the use of computer in education does not significantly improve students' achievement. There is a little robust evidence showing that the use of digital technologies (i.e. computers, internet connections and other digital software) for educational use among students leads to better achievement in mathematics and reading [1].

However, there are several research findings suggest the positive effect of digital technology in education. Reference [2], for instance, reports that some consistent findings indicate that digital technologies, such as calculators and computer software, improve student understanding and do not endanger student computational skills. Moreover, [3] in his extensive review studies shows that the use of digital technology in mathematics education leads to a significant positive effect (ranging from small to moderate effect size), especially for younger students (primary level or early secondary).

One of the most influencing factors determining the effectiveness of digital tools in education is the way the tools being utilised in classrooms. The way a digital learning tool is designed and employed in learning will determine the effectiveness of such a tool to student performance, especially in mathematics education [9] [10].
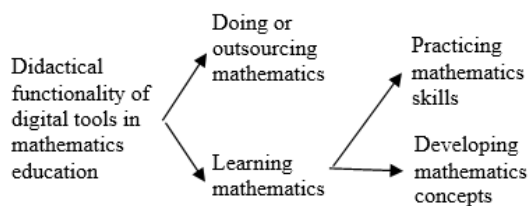

Fig. 1. The didactical functionality of digital technology in mathematics education (adapted from [8], [11], and [12])

Following [9] and [10], the role of digital tools in mathematics education can be classified into three types, namely tools for doing/outsourcing mathematics, tools for practising mathematics skills, and tools for developing mathematical concepts [8] [11] [12]. While the first role is less related to learning mathematics, the second role relates to learning mathematics, but the third role is considered

to have the most robust relationship to learning mathematics, especially for conceptual understanding (see Fig. 1). If digital tools are mostly employed for doing mathematics, such as using a digital calculator to assist students in calculation, it will lead to less conceptual understanding of mathematics. It is also true once the tools are mostly employed for practising mathematical skills, such as an online program for mathematical practice. It cannot guarantee to lead to better mathematics achievement [12]. Only once the tools designed for fostering concept development, it strongly promotes students conceptual understanding of mathematics, which consequently improves students achievement [7].

However, identifying the characteristics of the digital tools to foster concept development is the biggest challenge of researchers and educators in this field.

Following the challenge, many research has been conducted to investigate the characteristics of digital tools for promoting conceptual understanding of mathematics (e.g. [13], [14], [15], [16]). It stresses the importance of sensorimotor once children work with or through digital tools in order to promote the development of children's conceptual understanding of mathematics [7]. It is based on the claims that sensorimotor based activities shape the basis of cognition; therefore, it needs to root mathematical knowledge in bodily experiences. The underlying principles governing the sensorimotor based digital tools is then called as the embodied instrumentation. This idea stresses the importance of bodily experience with utilised digital tools to develop students' conceptual understanding once they learn in digital learning environments. In order to understand this notion, it is essential to grasp the framework underpinning the notion, namely the instrumentation theory and the embodied cognition theory.

III. THE INSTRUMENTATION THEORY

The instrumentation theory in this paper refers to the notion of the instrumental approach [17] [18]. It describes the interplay between the tools use (i.e. artefact) and users' cognitive development (i.e. schemes) in the learning contexts. The interplay can be generally stated through the claims that users' knowledge shapes the use of tools, and the constraints and the opportunities within the tools reshape users' cognitive development simultaneously. Therefore, a purposively designed tool may potentially lead to intended users' cognitive development.

In the instrumentation theory, a distinction between artefact and instrument is clearly defined [7] [17] [18]. The artefact is more or less the object that is used as a tool, for instance, a digital calculator, computer software, manipulative tools, and many more. This tool is not a necessary physical object, although mostly it is. The ways a user, for example, a student, interacts with the artefact to reach a specific

goal is called schemes. This scheme refers to [19] notion where it is considered as human cognitive structures in organising and processing information once they interact with and make sense of their environments. An instrument is created once an artefact interacts with its user under particular schemes performing a meaningful task. It implies that an artefact, together with its utilisation schemes, constitutes an instrument [7] [18].

In the instrumentation theory, an instrumental genesis is defined as the process in which an artefact becomes a part of an instrument in the hand of a user [20]. A reciprocal relationship between an artefact and a user is established during the instrumental genesis [18]. While the users' knowledge guides the way the tool is used (shaping the tool), the constraints of the tool shape the users' cognitive structures (shaping the schemes) and their emergent conceptions. For example, a student who uses a digital tool that provides a graphical visualisation of the solution of two linear equations will have a different conception about the meaning of the solution compared with a student who utilises a digital tool without such a display. Here, the constraints of a tool shape students' conceptions. Another example taking from the use of a digital calculator shows how users' knowledge shapes the way the tool is used. The student who does not recognise the bracket function in the digital calculator tends to calculate a simultaneous computation partially. These examples indirectly show the interplay between mathematics (i.e. the mathematical conceptions) and the tools use where the tools may shape mathematical practices and conceptions on the one hand, and the conceptions guide the functionality of the tools on the other hand. Once a user's knowledge shapes a tool or guides the way the tool is used, this process is called as instrumentalisation. Meanwhile, the instrumentation is the process once the affordances and the constraints of the tool influence the user's knowledge (e.g. problem-solving strategies) and the corresponding emergent conceptions. Both the instrumentation and instrumentalisation are the core ideas underpinning the instrumental genesis [7] [18].

The bilateral relationships between the tools and the users indicate the interplay between the artefact and the schemes. Since the schemes is an unobservable cognitive structure underpinning users' action, the systems can only be observed indirectly by inferring from the users' movements directed by the scheme. Such observable object is then called as the instrumented techniques. Here, the instrumented techniques are defined more or less as stable sequences of interactions between the artefact and the user with a particular purpose [18]. It can also be considered that the techniques are the observable pair of the invisible mental scheme.

Fig. 2. The broken calculator as a designed tool to foster students' conceptual understanding of numbers decomposition

Due to the reciprocal relationship between the artefact and the schemes, the instrumented techniques are guided, shaped and inspired by the users' knowledge (i.e. the schemes) and the constraints and opportunities of the tools use (i.e. the artefact). Both of them share considerable influences on instrumented techniques. Moreover, it is important to stress that since the tools shape the techniques, and they also reflect users' knowledge, the techniques potentially contribute to the development of the users' knowledge [7]. It implies that a purposively designed tool which leads to a particular instrumented technique may potentially foster users' knowledge and understanding.

An online digital tool called Broken Calculator (BC) is taken as an example to illustrate the interplay among the tools, the instrumented techniques, and the users' knowledge development. Suppose a user is challenged to determine the outcome of 4×12 employing the BC where only a few functions work (see Fig. 2). Since it is impossible to generate 4×12 using the given tool directly, the user may explore other potential strategies to tackle the problem. Utilising his understanding of the multiplicative relationship between 3 and 4 to generate 12, where 3×4 is 12, he may reach to the idea of transforming 4×12 into $4\times3\times4$ where this problem is solvable through the tools. In this example, the designed tool or artefact (i.e. the BC and the mathematical task) shapes the user's instrumented technique (e.g. decomposing 12 to be 3×4). Here, inspiring by his prior knowledge of the multiplicative relationship between 3 and 4 to generate 12 (i.e. user's knowledge or scheme), he comes up with the idea of transforming 4×12 into $4\times3\times4$. This technique consequently leads to the development of the user's understanding of the notion of number decomposition once dealing with multiplication problems (i.e. user's knowledge development or new schemes).
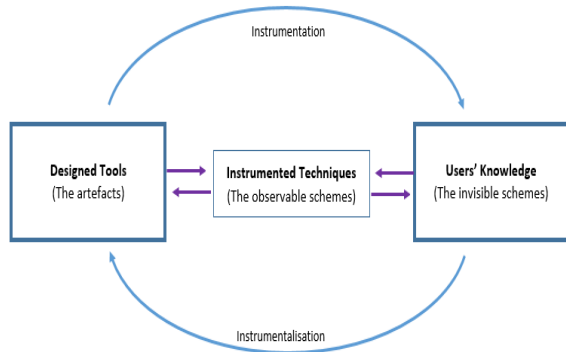
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Fig. 3. The complex bilateral relationships between tools and users in the instrumentation theory

Overall, the instrumental approach stresses the strong bilateral relationship between learning tools (i.e. the artefact) and users' cognitive development (i.e. schemes). Here, the tools could shape the users' knowledge (the instrumentation) and the users' knowledge influence the way the tools being orchestrated (the instrumentalisation) simultaneously. The instrumented techniques mediate the interactions between the tools and the users during the process of instrumental genesis (i.e. the simultaneous process of instrumentation and instrumentalisation). These complex relationships are described by Fig. 3. It implies that an intended cognitive development can be facilitated through the use of appropriate implementation of designed learning tools, and by the same time, the use of the tools is guided by the users' cognitive development. Hence, providing a supportive (effectively to reach goal) and appropriate (relevance with users' knowledge) learning tools lead to the intended users' cognitive development and skills. This theory, moreover, reminds us of the importance of instrumental genesis as a path to learning mathematics, which is a crucial step forward to fostering learning while employing digital tools.

IV. THE EMBODIED COGNITION THEORY

Cognition cannot be regarded as an exclusively mental affair, but it is the product of human corporal experiences taking place in interactions with the physical and social world. (e.g., see [21] [22] [23]). In line with this view, reference [24] asserts that human cognition is influenced by the capabilities and limitations of their body since it is the product of the interplay between their perceptual system (i.e. perception) and their physical skills (i.e. body skills).

Cognitive structures or schemes are created and developed by the human being in efforts to understand the complexities of their world through the process of assimilation and accommodation as their perceptual and physical dimensions (i.e. sensory and motoric skills) interact reciprocally to make sense of their world [19] [25] [26]. In other words, human perceptions and their sensorimotor skills (e.g. kinesthetic, visual and audio skills) are the interplay

tools to grasp a conceptual understanding of their world. In this case, a sensorimotor experience is one of the expertise in constructing human cognitive structures.
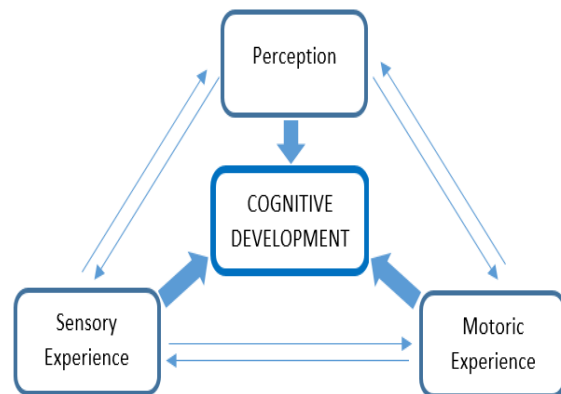

Fig. 4. The interplay among perception, sensory and motoric experiences in shaping human cognition

Although Piaget asserts that sensorimotor is the mechanism of learning in the early stages of human life, it is also relevance for all stages since they share similar cognitive challenges, trying to understand new phenomena by employing human perceptual and motoric skills (i.e. sensorimotor). Once an adult faces a new phenomenon, for instance, he will utilise his perception together with his sensory and motoric skills to understand the phenomenon. Here, bodily experiences (i.e. sensory and motoric experiences) contribute to the development of human cognition.

The scheme or cognition structure development involves the intertwined development of sensorimotor skills and cognition [7]. In line with Piaget's view, the sensorimotor experiences, which are based on bodily experiences, shape the cognitive structure. This fact leads to the notion of the embodied nature of cognition, where body-based experience significantly contributes to the development of human cognition (see Fig. 4).

Although it is considered a highly abstract and mental entity, mathematics cognition is acknowledged to be rooted in sensorimotor activities, and therefore, mathematical objects are grounded in sensorimotor schemes [7]. Following such a paradigm, Drijvers in [7] promotes the notion of embodied instrumentation. Such idea stresses the importance of bodily experience (i.e. the sensorimotor experience) to develop students' conceptual understanding of mathematical concepts once they learn in digital learning environments. It is based on the claims that sensorimotor activities form the basis of cognition; therefore, it needs to root mathematical knowledge in bodily experiences.

The embodied nature of cognition is the primary basis of the embodied cognition theory. The theory asserts that the sensorimotor experience which is

integrated a body-based experience shapes the basis of human cognition, including the mathematical cognition.

V. SENSORIMOTOR BASED DIGITAL TOOLS

The notion of the instrumentation theory, together with the embodied cognition theory, inspires the design and the development of digital tools in mathematics education. While the instrumentation theory asserts the interplay relationship between tools and users' cognitive development, the embodied cognition theory stresses the reciprocal relation between sensorimotor experience and human cognition. These theoretical foundations lead to the argumentation that tools (i.e. digital tools), which allow sensorimotor experiences, is more promising to foster human (e.g. children) cognitive development. This argumentation is the basis of the notion of the sensorimotor based digital tools in education, elaborated in this paper.

An example is presented here to understand the notion. It is taken from a digital tool developed by Alberto et al. on the project called Mathematics Imagery Trainer – Trigonometry (MIT-T) [27]. Fig. 5 shows a brief illustration of the tool, and it can be traced in https://youtu.be/1eOU4XyyHmg for more details. The tool is designed to work with touch screen devices. Two main objects appeared on the screen. The first object shows a unit circle, and the second object represents a sine graph. A movable correspondence point is on each of them. A colour-change rectangle is placed as the frame of the two objects. Its line colour will turn to be green once the movable point in each object is corresponding, that is if the two points are at equal height. Once the rectangle turns to be green, it implies that a correct match is made between the sine of an angle in the circle and the function value of the sine in the graph. For example, Figure 5 shows that sine of 150 degrees (showed in the unit circle) is 0.5 (appeared in the sine graph). However, these corresponding values do not appear in the tool. It is something that the users required to discover.
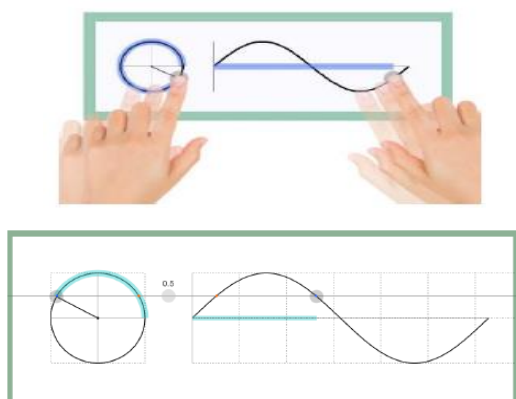

Fig. 5. Mathematics Imagery Trainer – Trigonometry (MIT-T) (source: [7])

While working with the tool, users are asked to move the movable points simultaneously such that the rectangle is green. They keep moving the two points while preserving the green colour of the rectangle. In the end, they are expected to realise the rule governing the rectangle to be green and understanding the relationships between the sine of an angle in the circle and the function value of the sine in the graph.

To come up with such understanding through the tool, the users will experience a sensorimotor experience to keep the two points on the same height. Moreover, the colour change on the rectangle concerning the point movement provides another sensorimotor experience that is the eye-tracking to identify the relationship between the point movement and the change of the colour. It is an example of how sensorimotor based digital tools shapes and develops users' cognition.
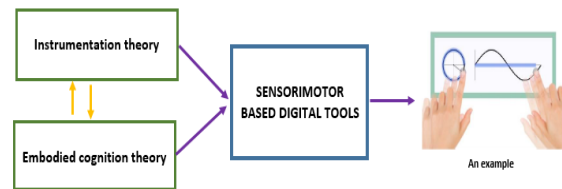

Fig. 6. The mutual relationship between the instrumentation theory and embodied cognition theory to produce sensorimotor based digital tools

MIT-T is an example where the instrumentation theory works together with the embodied cognition theory to produce more promising digital tools for learning called sensorimotor based digital tools (Fig. 6). These tools offer rich experiences to their users, including perceptual, sensory, and motoric experiences. These experiences foster human cognition development since they are in line with the nature of human cognition development.

VI. CONCLUSION AND FUTURE CHALLENGES

Regarding the controversy on the benefit of digital technology in mathematics education, what and how a digital tool is presented to students is the matter determining the effectiveness of such a tool. This paper suggests that digital tools that offer comprehensive experiences (i.e. perceptual, sensory and motoric experiences) to their users foster students' cognitive development. Such a tool is appropriate, especially for learning abstract and mental entities, such as mathematics, for its power to embody abstract concepts. Therefore, sensorimotor based digital tools are recommended to be implemented in the mathematics classroom.

However, implementing such a tool in classrooms comes with several challenges. What are the characteristics of the classroom environments and learning principles to support the implementation of sensorimotor based digital technologies? What the socio and socio-mathematical norm are required?

What is the role of teachers? How will the learning tasks and activities be looked like? Those are several questions arising which is call for future research.

Moreover, the implementation of sensorimotor based digital tools in classroom calls for collaborative work among educators (mathematics teachers), mathematics education researchers, and ICT experts. While teachers focus on carrying out the tools in the classrooms, researchers and ICT experts concentrate on designing and developing the tools and promising learning tasks and activities associated with the tools to reach expected learning goals.

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