

# Embodied Task to Promote Spatial Reasoning and Early Understanding of Multiplication

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This study enquires into the embodied processes of children in solving multiplication tasks, considering how such processes can expand access to spatial reasoning skills and simultaneously develop students' understanding of multiplication. The analysis focused on four Year 2 students as they completed two embodied tasks. The aim was to understand how embodied tasks could stimulate students to use spatial reasoning to explore and understand multiplication as equal groups in array forms. The findings suggest that engaging students with embodied tasks stimulate them to think about mathematics spatially and reflect on their thinking about mathematical ideas.

## Introduction

Although numerous research findings strongly confirm the intense link between spatial reasoning and mathematics competency (e.g., Barmby et al., 2009; Gunderson et al., 2012), less study has been conducted focusing on how spatial reasoning can be used to facilitate mathematical understanding (Lowrie et al., 2020; Newcombe, 2018; Newcombe et al., 2019). As spatial reasoning is naturally embodied (Thom et al., 2015; Thom & Hallenbeck, 2021), one way to promote spatial reasoning in mathematics learning is through engaging students with embodied tasks (i.e., the tasks that stimulate physical and sensory systems). The link between spatial reasoning and embodied experiences is described by Thom et al. (2015) in the following assertion, "A child who engages in the act of mentally rotating a shape is not just performing an act of spatial reasoning. She is demonstrating embodied mathematics. She is recursively enacting her embedded and embodied knowing (p. 81)." Here, spatial reasoning is about how bodies regularly sense and make sense of the situations of the physical world.

Therefore, the potential of the embodied task as a learning strategy is grounded on the theory of embodied cognition. The theory claims that our knowledge or cognition is shaped by the experience of our physical and sensory systems within our environment (Shapiro, 2019; Shvarts et al., 2021; Varela et al., 2016). The embodied theory highlights two important points; first, cognition is contingent on the types of experiences gained by having a body with different sensorimotor abilities; and, second, these individual sensorimotor abilities are themselves inherent in the larger biological, psychological and cultural contexts (Varela et al., 2016).

Regarding the relationship between spatial reasoning and embodied activities and their potential to promote mathematical understanding, this study aimed to understand how embodied tasks stimulate students' spatial reasoning and encourage the students to use this reasoning to explore and understand mathematical concepts. To reach the goal, learning tasks involving embodied experiences were designed to develop students' awareness of the idea of multiplication as equal groups (e.g.,  $2 \times 3$  as two groups of 3) through array structuring tasks. The concept of multiplication was chosen for three reasons. First, the idea is foundational in early mathematics since it is the groundwork for proportional thinking used in real-world applications (Fosnot & Dolk, 2001). Second, although it is foundational, many students retain a poor understanding of the concept as they are mostly taught multiplication as a list of mathematical facts to be memorised without understanding the underlying mathematical concepts (Hendriana et al., 2019). Third, the underlying concepts of multiplication can be discussed in terms of space as the concepts can be effectively represented and communicated

through several spatial representations, such as number lines, bars or arrays (Kosko, 2019). Thus, this study aimed to address the question,

*How could embodied tasks stimulate students to use spatial reasoning to explore and understand multiplication as equal groups?*

## Embodiment, Spatial Reasoning and Understanding of Multiplication

The fundamental claim of embodied cognition is that the features of our cognition are shaped by the aspects and the experiences of our body with the environment (Shapiro, 2019; Varela et al., 2016). In this theory, knowledge is developed as the result of the sensory and motoric experiences of our body with the external world, such as learning tools, through a mechanism called perception-action loops (Shvarts et al., 2021). As students interact with a learning tool, the perception-action loops can be described as a simultaneous interaction between body and mind in producing knowledge. Here, the initial perceptions toward the learning tool guide users' actions on the learning tool and, at the same time, the actions generate verified or extended perceptions toward the learning tool (Shvarts et al., 2021).

Moreover, in the embodied cognition theory, an embodied action can be viewed as the extended visible or tangible form or a concrete example of conceptual understanding (de Freitas & Sinclair, 2013; Thom et al., 2015; Thom & Hallenbeck, 2021). Thom et al. (2015) coined this action as an observable knowing where students' understanding is reflected by or represented through their physical actions. For example, when an individual student arranges unit cubes into several equal rows, this student might be described as embodying the concept of the equal group of multiplication. When a student shows that  $a \times b = b \times a$  by rotating  $a \times b$  array to form  $b \times a$  array, this student may be representing the embodiment of the commutative nature of multiplication. Thus, the embodied cognition theory suggests that students' experiences need to be situated in designed embodied tasks that support them to develop the intended understanding. It follows that student understanding can be assessed by investigating their physical actions in situated conditions. Therefore, in this study, we define an embodied task as a task that requires physical actions to perform purposive actions under a situated condition.

Embodiment and spatial reasoning are strongly related and even significantly overlap. Spatial reasoning is embodied since spatial reasoning arises from making sense of the embodied sensorimotor experiences (Thom et al., 2015; Thom & Hallenbeck, 2021). For example, in the context of mathematics, a child who is mentally rotating a shape is not only performing an act of spatial reasoning but also demonstrating embodied mathematics (Thom et al., 2015). An individual (whether a child or an adult) mentally manipulating spatial properties of an object using gesture, movement, drawing, modelling, and so on, with or without signed/spoken/written language, is simultaneously demonstrating embodied mathematics (Thom & Hallenbeck, 2021). Therefore, embodying mathematical ideas in physical movements will foster the use of spatial reasoning as such reasoning is stimulated once a student navigates his/her body in space during the movements.

The array is recognised as a powerful spatial representation that allows access to several big ideas of multiplication, such as equal groups and the binary nature of multiplications (Barmby et al., 2009; Battista et al., 1998; Kosko, 2019). The relationships among embodiment, spatial reasoning and the understanding of multiplication can be demonstrated by using spatial tools, such as arrays, to explore, practice and communicate mathematical ideas underlying multiplication. For example, a student who is structuring arrays to reason about multiplication is considering embodied events as the spatial information embedded in the array (e.g., shape, size, location, and distance) is oriented, moved, or managed by the potentials of our body (sensorimotor capacities). Here, in relation to the concept of the body in/of mathematics (de

Freitas & Sinclair, 2013), the ideas of multiplication are animated and practised through and by body experiences of structuring arrays. Such an embodied experience intensely fosters spatial reasoning as it involves spatial structuring thinking that facilitates students to develop a meaningful understanding (Battista et al., 1998).

## Method

This study aims to develop embodied learning tasks that stimulate students' spatial reasoning and promote students' awareness of multiplication concepts as equal groups of objects. Therefore, design research was employed consisting of three phases, namely design preparation, teaching experiments, and retrospective analysis (Gravemeijer & Cobb, 2013).

During the design preparation, a literature review was conducted to form the basis for designing the embodied tasks. The embodied tasks were designed to help students develop early awareness of multiplication as equal groups. Two related embodied tasks were formulated. Task 1 invited students to explore different ways of counting 12-unit cubes and used the cubes to represent their counting strategies. Here, it is expected that the students will develop several counting strategies, such as counting by twos or threes and represent the strategies in arrays. By arranging cubes in arrays, it is expected they will see the group structure of the cubes in arrays. As the follow-up, Task 2 asked students to imagine several arrays consisting of 12-unit cubes and then draw the imagined arrays on a grid paper. It is expected that the students will be aware of the group structure of multiplication while imagining and drawing the arrays.

In the next phase, the teaching experiments, the designed embodied tasks were tested in the classroom setting involving eight students (four Year 2 students, two Year 3 students, and two Year 4 students). However, for the current paper, the analysis focused on the findings from the Year 2 students (50% girls) as the data produced by those students best exemplify how embodied learning tasks could stimulate students' spatial reasoning and promote students' awareness of the concepts of the equal groups of multiplication. During the experiments, the teaching-learning activities were observed directly, and video recorded. As well, students' written work on display boards and worksheets were collected.

In the final stage, the retrospective analysis, students' embodied responses toward the tasks together with the relevant written works were analysed for two purposes, namely (1) to gain an understanding of the relationship between the embodied tasks and students' responses, and (2) to understand how the embodied tasks promote the intended understanding. The retrospective analysis was conducted task by task chronologically to see how each task stimulated students' responses. For this paper, the analysis focused on how the two embodied tasks stimulated students to think of and reflect on the array structure about equal groups in multiplication.

## Results and Discussion

### *Students' Responses on Task 1*

The first task asked the students to explore different ways of counting 12-unit cubes and used the cubes to represent their counting strategies. The data showed that three out of four students initially counted the unit cubes one by one. But, after they were asked to count them differently, they developed various counting strategies, such as counting by twos or threes, and represented their counting by arranging the cubes in arrays (see Figure 1). Student A, for example, initially counted the cubes by threes before she decided to count them by twos, where she made six groups of two in an array. Student B's first attempt was counting by threes and arranged the cubes in an array model representing four groups of three.

Meanwhile, Student C, at the first attempt, counted the cubes by twos, connected their cubes, and then arranged the unit cubes to form three groups of four. Student D initially counted the unit cubes by fours and then by twos. In contrast with Student B, who arranged their unit cubes horizontally or in rows, Student D arranged them vertically in columns.

Moreover, it is identified that the students glanced over at other students' array constructions to get inspiration of the array structure from others. For example, as Student D was attempting to make another unique array, he quickly constructed a  $2 \times 6$  array for his second attempt after seeing Student A, who made a  $6 \times 2$  array. He recognised that six groups of 2 can be represented as two groups of 6. As the students arranged the unit cubes in arrays, the teacher used this opportunity to examine whether the students could see the structure of the groups in an array. During the discussion, the students conveyed that 12 cubes could be represented in various grouping forms, such as two groups of 6 or three groups of 4. The teacher used the students' production of group structure as the context to introduce the notion of multiplication as equal groups. For example, six groups of 2 can be written multiplicatively as  $6 \times 2$ .

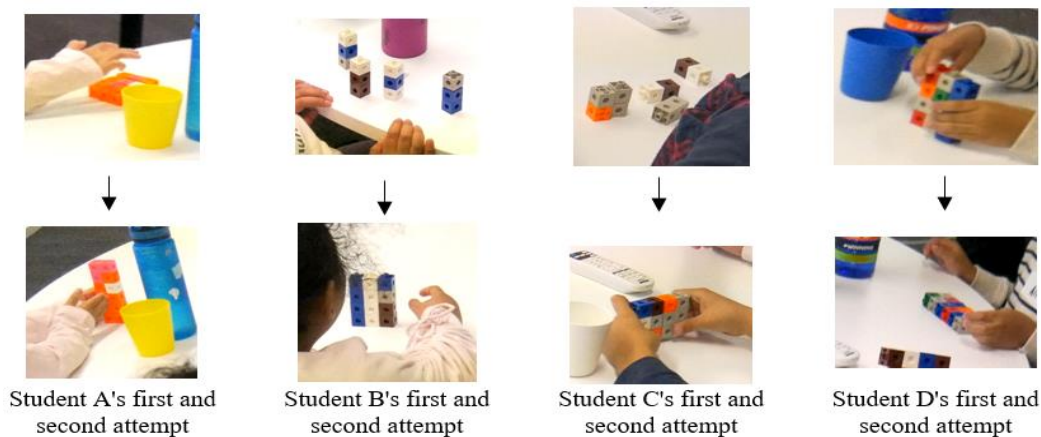


Figure 1. Several students' self-constructed arrays to track their counting.

### Students' Responses on Task 2

In Task 2, without cubes, the students were asked to imagine several arrays consisting of 12-unit cubes and then draw the imagined arrays on paper. Through the task, it is expected that the students will be aware of the group structure of multiplication while imagining and drawing the arrays. The data showed that the embodied task of drawing the imagined arrays stimulated the students to validate their conjecture of the expected array structure. For example, Student A intended to draw the array for two groups of six cubes (see Figure 2). She initially drew a big rectangle and split the rectangle vertically by drawing six vertical partition lines. She then drew a horizontal partition line splitting the rectangle horizontally into two sections and counted the number of unit cubes on the first row, where she got seven-unit cubes instead of six. Upon realising this mistake, Student A erased one vertical partition line to make six columns.

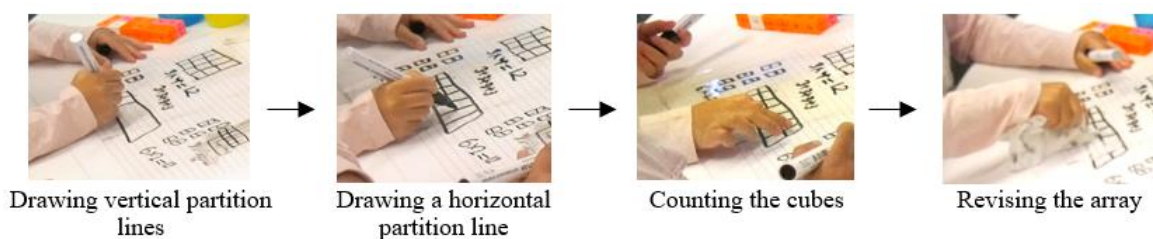


Figure 2. Student A's experience of reconstructing a  $2 \times 7$  array into a  $2 \times 6$  array to represent two groups of 6.

Similar to Student A's experience, Figure 3 shows that Student C intended to draw six groups of two. He modified a  $6 \times 3$  array into a  $6 \times 2$  array because he realised that the  $6 \times 3$  array does not represent six groups of two as he expected. Initially, he drew a big rectangle and split the rectangle into two sections vertically, then split the rectangle horizontally by drawing horizontal lines forming six rows. Next, to draw two columns, Student B drew two vertical partition lines, although he instead attained three columns. He suddenly became aware of his mistake after counting the generated unit cubes (the square cells) by twos (row by row), where he found 18 cubes instead of 12. Immediately, he removed the last column creating a  $6 \times 2$  array.

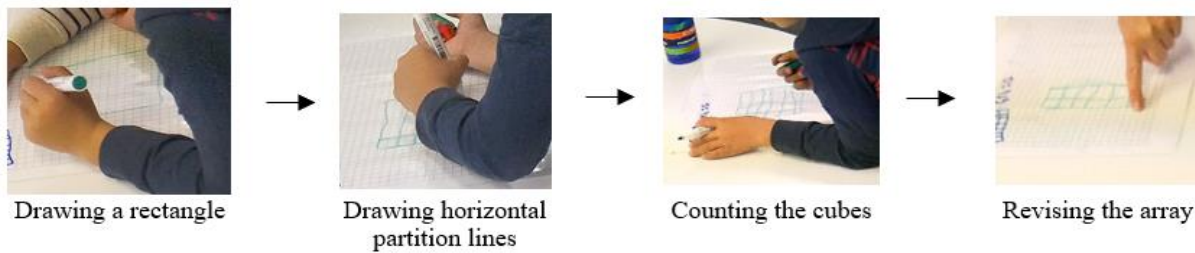


Figure 3. Student C's experience of reconstructing a  $6 \times 3$  array into a  $6 \times 2$  array to represent six groups of 2.

It was identified that Students A and C's mistakes were similar. They drew one more column than what they were intended to have. As they drew the number of vertical partition lines equal to the intended number of columns, they may have thought they had already drawn the correct number of columns. In fact, drawing vertical partition lines generate  $n+1$  columns. For example, drawing two vertical partition lines generate three instead of two columns. As the students could do self-assessment and correction simultaneously, they utilised their spatial reasoning on the spatial visualisation of the group structure. Such an experience contributed to their conceptual understanding of the structures.

## Discussion

Regarding the intimate link between spatial reasoning and embodied activities and their potential to promote mathematical understanding, this study aimed to understand how embodied tasks stimulate students' spatial reasoning and promote mathematical understanding. The impact of the embodied tasks was sought through discussing two key findings. First, the embodied tasks in the learning stimulated students to think of the equal-group structure of multiplication. Second, the tasks stimulated them to reflect on their thinking of the mathematical concept.

### *Embodied Tasks of Structuring Array Stimulating Students to Think of the Equal-group Structure of Multiplication*

The findings from students' responses on the first task show that the students' physical actions of arranging and rearranging the unit cubes in several different group structures express what they have in their minds about the spatial structure of the array. Considering the idea of the body in/of mathematics (de Freitas & Sinclair, 2013), the actions serve as the extension of their thinking of the array structure as they acted purposively to express their thinking of the structure. Therefore, the actions themselves can be regarded as the representation of their understanding of the structure. Their ability to construct 12-unit cubes in several arrays shows that they understood the underlying array structures where 12 can be represented in various ways of grouping, such as three groups of four, two groups of six, or six groups of two.

Furthermore, the array is used as a spatial tool to represent the group structure. Students' actions through the spatial tool consequently stimulate the use of spatial reasoning where, for

instance, the students need to consider the changes in the group structure of the array as the result of moving or modifying unit cubes. Thom et al. (2015) consider this kind of action as observable knowing of which and by which the students' spatial reasoning grows. For example, as each student was asked to construct a unique array from the 12 cubes, Student D looked and imitated the  $6 \times 2$  array made by Student A to construct a  $2 \times 6$  array by mentally twisting the  $6 \times 2$  array. Student D could differentiate the group structure between the  $6 \times 2$  and  $2 \times 6$  array due to the rotation.

The embodied actions of structuring arrays and their reasoning of the spatial structure of the arrays interplay simultaneously during the process of constructing the meaning of the group structure. On one side, the embodied actions stimulated the students to activate their spatial reasoning and serve as the way to clarify their mathematical reasoning. For example, once the students develop their conjecture that the 12-unit cubes can be represented in four groups of three, the actions of arranging the unit cubes into four rows of three will verify the conjecture. On the other side, and at the same time, spatial reasoning navigates students' embodied actions of arranging and rearranging the array structure. For example, bodily reconstructing a  $6 \times 2$  array into a  $2 \times 6$  array requires the students to envision the spatial changes on the  $6 \times 2$  array; as a result, the transformation and how the changes generate the new spatial structure. Here, the simultaneous interactions between the perceptions of the spatial changes on the array (as the result of the embodied actions) and the body actions on the array (as the result of the spatial perceptions) create perception-action loops (Shvarts et al., 2021). In this context, the perception is generated by the spatial information of the array, and the embodied actions are triggered by the perception. By considering Shvarts et al. (2021) theory of embodied instrumentation, the perception-action loops are the generator of knowledge and understanding where the progressive and simultaneous interaction between spatial perceptions and the embodied actions on the array generate understanding of the group structures underlying the array.

### *Embodied Tasks of Drawing Arrays Stimulate Students to Reflect on Their Thinking About the Group Structure*

Although understanding the concept of equal groups of multiplication is known to be challenging for many students (Battista et al., 1998), Students A and C's responses on Task 2 suggested that the embodied experience of structuring arrays by drawing the array has the potential to promote students' awareness of the group structures. As the students were engaged in the embodied tasks, they could develop an awareness of the concept of the equal group and how the concept can be expressed on arrays. They understood that they had to modify their previous array to get the intended array and realised that parts of the array needed to be revised to attain the correct one. Students also recognised that, as they modified the arrays, they must preserve the same number of unit cubes in each row or column. Such awareness reflected their understanding of the array representation of multiplication.

Moreover, their embodied spatial experience of structuring the array and the array itself (the representation of the groups in rows and columns) allowed them to reflect their thinking, for example, thinking whether they created the array for 12-unit cubes or not. By drawing the array, they animated the virtual ideas of multiplication (i.e., equal grouping) through the embodied experience of drawing. Looking at the notion body in/of mathematics (de Freitas & Sinclair, 2013), their experiences of recognising incorrectness through the visualisation of the array and bodily modification of the array reflected their conceptual understanding of how arrays can be used to express the abstract ideas of equal groups of multiplication.

At the end of the lesson, the students' conceptual understanding of equal groups of multiplication could be asserted as they represented the concept of equal groups in various ways, such as through arrays and repeated additions, which helped them define the product of

multiplication (see Figure 4). Student A, for example, defined  $3 \times 6$  as the sum of 3 groups of 6, which can be represented as an array having three rows of 6.

The analysis of the tasks suggested that the mechanism underlying the mutual connections among the three constructs (i.e., embodied tasks, spatial reasoning, and multiplications) can be explained through the framework of embodied instrumentation proposed by Shvarts et al. (2021). In the embodied instrumentation, the interaction between body capacities (sensory and motoric skills) and the situated learning condition (e.g., learning multiplication through physically structuring and drawing arrays) generate knowledge or understanding resulting from progressive perception-action loops. The perception-action loops are simultaneous interactions between body and mind in producing knowledge (Shvarts et al., 2021). In the context of the current study, perception was generated by the ability of the body's capacities (sensory and motoric skills) to see, hear, or become aware of the spatial structure of arrays to represent the equal groups as the result of bodily modifying the array. Meanwhile, the action was the act of bodily modifying the array, which is stimulated or guided by the perception. As the perception-action loops were continuously verified and refined through time simultaneously, the perception-action loops generated the intended verified knowledge and understanding (Shvarts et al., 2021). For example, it was identified that the students initially did not recognise the notion of equal groups of multiplication. However, as the students intensively interacted with the spatial tools situated by the embodied tasks, they began to develop that awareness. Throughout the process, spatial reasoning played significant roles both as the mediator between the perceptions and the actions and as the catalyst that stimulated progressive perception-action loops. As the students worked with the spatial tool to represent mathematical ideas, spatial reasoning facilitated them in constructing appropriate perceptions about the mathematical ideas spatially. At the same time, their body actions were guided by their spatial reasoning as they are dealing with a spatial environment (e.g., spatial tools) to communicate mathematical ideas. In the context of the current study, where spatial tools are predominantly used to express mathematical ideas, having a good sense of space may foster the development of perception-action loops.

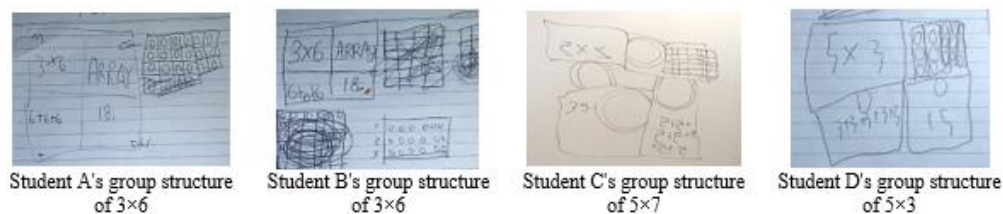


Figure 4. Students' self-constructed representations for several multiplications.

## Conclusion

The students in this study demonstrated the connections among embodied actions, spatial reasoning, and mathematical understanding. Their embodied actions were portrayed through the embodied experience of structuring and drawing arrays. Meanwhile, the use of spatial reasoning was observed through the use of a spatial tool (the array) to communicate and express mathematical ideas of multiplication. Finally, the progression of their mathematical understanding was reflected by their ability to define multiplication as equal groups in the form of array structure and repeated addition.

The findings of the current study highlight two important points. First, engaging students with embodied tasks potentially stimulates them to think of and reflect on mathematical ideas spatially. The embodied tasks promote spatial reasoning to explore, communicate, and make sense of mathematical concepts spatially. Here, the embodied experiences and the spatial experiences provide the contextual meaning for the explored mathematical ideas. Second, the

use of embodiment theory should be considered in task design. The embodied-based tasks foster the formulation of knowledge and understanding through the mechanism called perception-action loops. In this mechanism, actions stimulate an understanding and, at the same time, understanding guides action. Progressive perception-action loops allow students to develop their thinking and reflect on their thinking of the situation and ideas being explored.

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