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The didactical use of multiple pictorial representations: The case of fractions division

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Introduction

- The role of representations in mathematics teaching and learning is overwhelming. It deepen students' conceptual and procedural understanding and is tools for problem-solving (NCTM, 2014)
- The use of representation is one of the key aspects in effective mathematics teaching and learning (NCTM, 2000; NCTM, 2014)
- Many studies unravel that the use of representations facilitate and support students develop mathematical knowledge (e.g., Pape & Tchoshanov, 2001; Webb, Boswinkel, & Dekker, 2008; van Galen & van Eerde, 2013; Sokolowski, 2018; Wahyu, 2021)
- Additionally, when students are able to use multiple representations or linking across representations in solving mathematics tasks, they demonstrate deeper understanding and enhanced problem-solving abilities (Fuson, Kalchman, & Bransford, 2005)
- This presentation discusses the use of multiple pictorial representations in teaching and learning of fraction divisions.

Theoretical perspectives

Conceptualizations of fractions division

- **Measurement**, **partitive**, **unit rate**, **the inverse of an operator multiplication**, and **the inverse of a Cartesian product** (*Sinicrope et al., 2002*)
- **The first three** are mostly taught in the classroom and presented in the text books (*Wahyu & Mahfudy, 2018*)
- The three have peculiar characteristics with respect to ***components*** (dividend and divisor), ***typical situation*** (e.g., fair-sharing), ***solution process*** (iterating or partitioning), and ***developed algorithm***
- This presentation draws on students' works on measurement and partitive fractions division

Theoretical perspectives

Multiple representations in fractions

- *"…**Mathematical representations** are visible or tangible productions … that encode, stand for, or embody mathematical ideas or relationships…Such representations are called external – i.e., they are external to the individual who produced them and accessible to others for observation, discussion, interpretation, and/or manipulation…"* (Goldin, 2020; p.566)
- In general, there are six classification of representations; contextual, visual, verbal, physical, and symbolic (Lesh, Post & Behr, 1987).
- Charts, tables, diagrams, models, computer graphics and formal symbol systems are examples of the external representations (Janvier, Girardon & Morand, 1993).
- Pictures are part of visual representations, called as pictorial representations (PR) in this presentation.

Theoretical perspectives

Multiple representations in fractions

- Three common PR used to denote fractions are number lines, area model and sets of objects. These are also called as models (Petit et al., 2016).
- Each PR has unique characteristics regarding the whole, equal parts and fraction (Figure 1, Petit et al., 2016; p.10).
- The students' works in this presentation include number lines, area model and/or sets of objects (multiple PR)

	The whole	*"Equal parts" are defined by*	*What the fraction indicates*
Area model	Determined by the area of a defined region	Equal area	The covered part of the whole unit of area
Set model	Determined by definition of what is in the set	Equal number of objects	The number of objects in the subset of the defined set of objects
Number line	Unit of distance or length (continuous)	Equal distance	The location of a point in relation to the distance from zero with regard to the defined unit

Figure 1. Features of visual models to represent fractions

Theoretical perspectives

Context-based tasks

- The use of context embedded in mathematics tasks is one of the tenets of realistic mathematics education (Gravemeijer, 1994)
- In PISA, context-based tasks or also called contextual tasks are mathematical problems presented within a *situation,* which can refer to a real world or fantasy setting, can be imagined by students, and can include personal, occupational, scientific, and public information (OECD, 2003)
- In this presentation, the context of the tasks given to students were real world (extra-mathematical context, OECD, 2003) or relevant and essential context (De Lange, 1995)

Research context

- This presentation is drawn from a project aimed at developing 5th grade students' conceptual understanding on fractions division (FD)
- The project follows realistic mathematics education theory (Gravemeijer, 1994) and was guided by design research (Bakker, 2018)
- HLTs were designed for five lessons comprising five tasks for group discussion; four tasks were referred to measurement FD and type I-IV FD (Schwartz, 2008) and three tasks for individual assignment.
- Before teaching experiments, some preparations were made to adjust the classroom context, for example, introducing multiple pictorial representations to represent fractions to the students. The students were not used to start lessons with discussing and solving context-based tasks.
- Two cycles of teaching experiments were carried out, including 6 students and 28 students in the first and second cycle, respectively.
- In this presentation, students' works on selected tasks are presented to understand the use of multiple pictorial representations in solving fractions division tasks

The tasks

Context	FD content	Pictorial representations (PR)	Notes
Mountain tracking (Individual task)	Measurement FD, type I (4 ÷ 2/3)	Area model to number line	The students were not given the representations to solve the task (self-developed)
Serving break for teachers (Group task)	Measurement FD, type II (2/3 ÷ 1/6)	Sets of objects to area model	Idem
Cake sharing (Individual task)	Measurement FD, type II (3/5 ÷ 1/5)	Area model to number line	Idem
Chocolate for outstanding team (Group task)	Measurement FD, type III (9/10 ÷ 1/5)	Area model to number line	Idem
Math assignment (Group task)	Partitive FD (3/4 ÷ 5)	Sets of objects to area and number line	The students were given the alternative PR to solve the task
Cake sharing (Individual task)	Partitive FD (5/6 ÷ 5)	Area model to number line	The students were not given the representations to solve the task (self-developed)

Findings

Math assignment task (Partitive FD)

Dwi has 3/4 hour to solve 5 problems in a math assignment. If she uses equal time for each problem, how many hours can she give to each?

- It was conjectured and proved that the students could not directly draw PR to represent the context 'hour'
- After some trials with (a), where 9 dots (9/12) were not divisible by 5 problems, the (b) was used; 15 stars were divisible by 5 problems.
- The students were struggling to determine 3/4 of the 20 stars (understanding equal parts that represent 3/4, also the case of equivalent fractions)

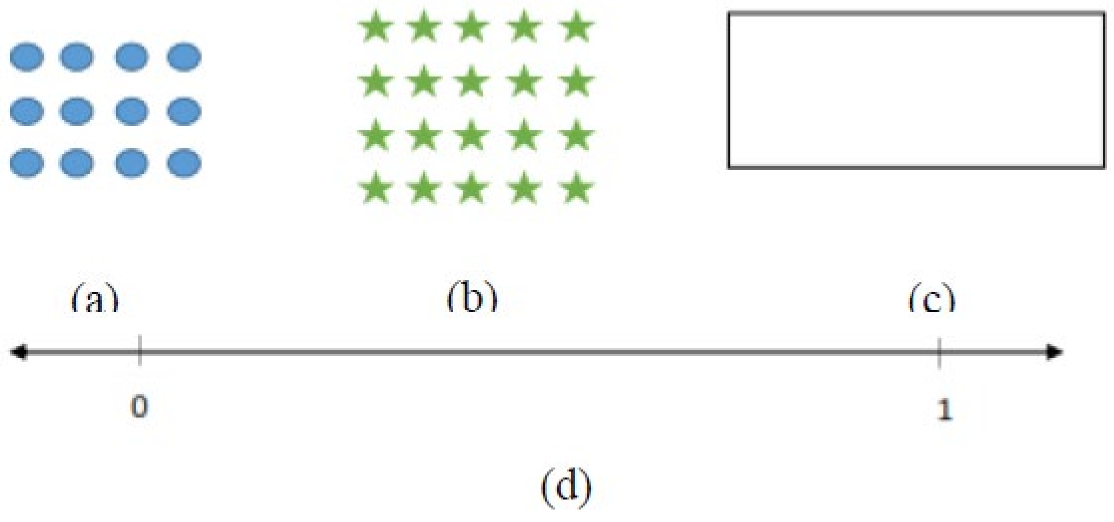


Figure 2. The suggested PR for the task

Findings

Math assignment task (Partitive FD)

- Were they struggling to determine 3/4 in number line? With the given number line, the answer was yes. Determining the equal parts (3/4 or 15/20) within range 0-1 was not easy for primary students
- Defining 3/4 or 15/20 in a area model was not difficult for the students after working with the sets of stars; each star represents each block in the rectangle

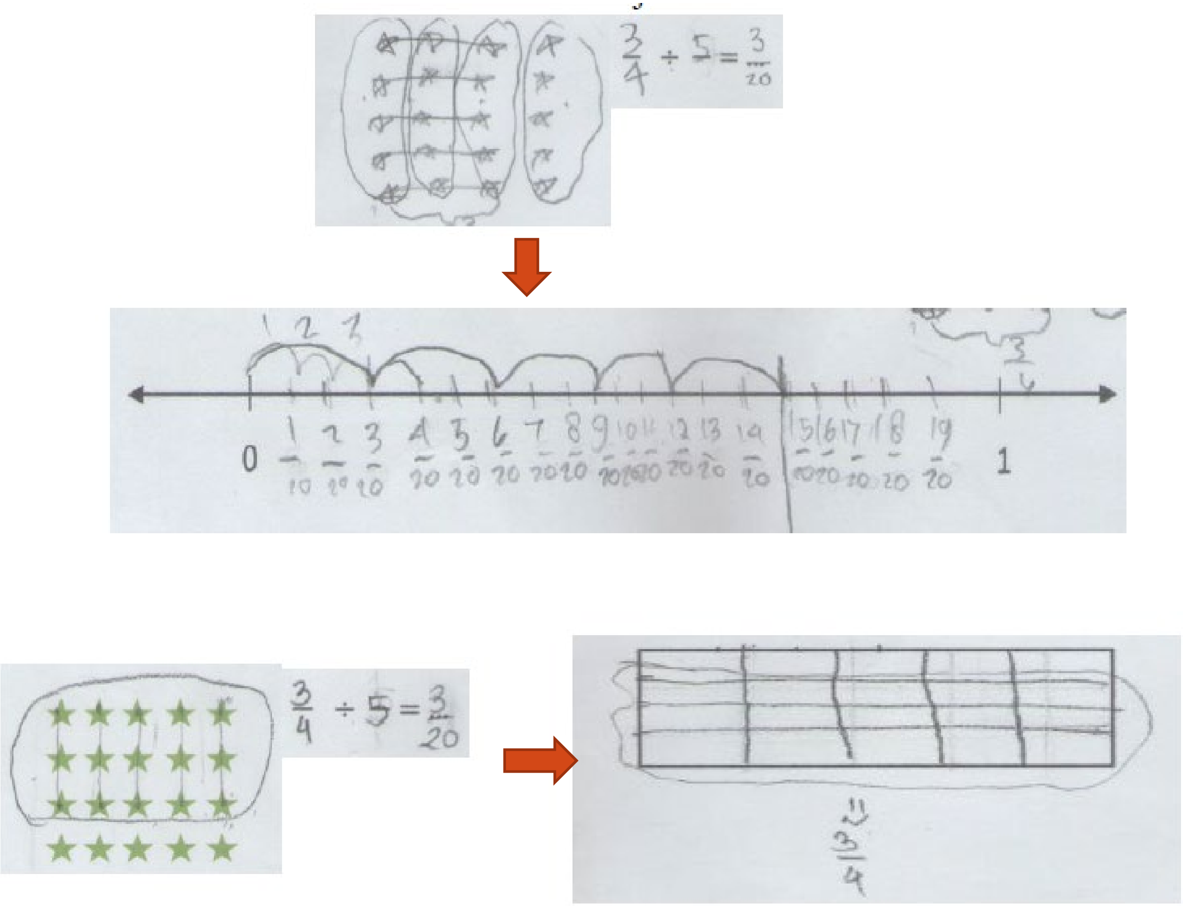


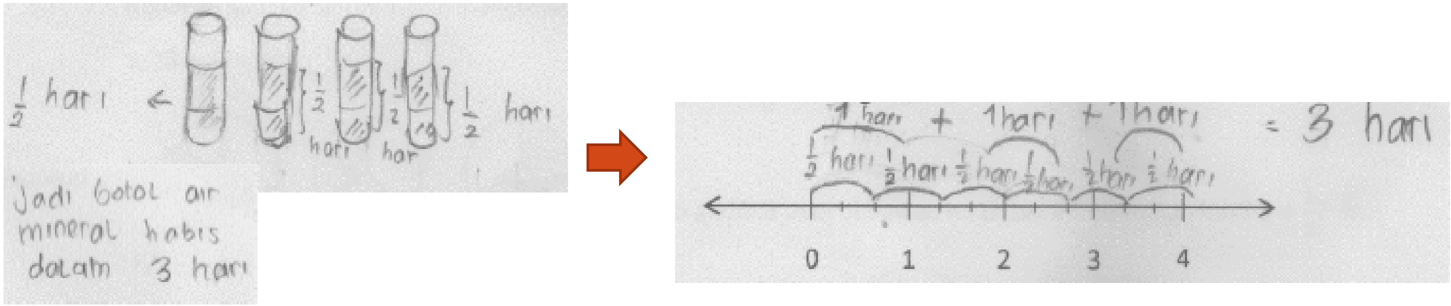
Figure 3. Linking sets of objects to number line and area model

Findings

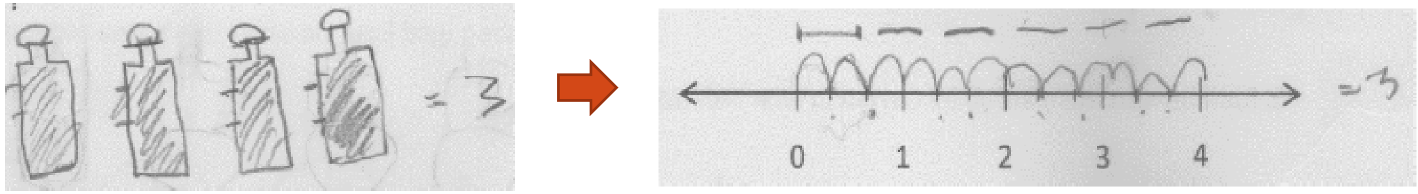
Mountain tracking (Measurement FD)

Teguh goes for a mountain tracking. He brings 4 litres of water and drinks 2/3 of a bottle each a half day. How many days can 4 litres of water lasts for?

- Determining and translating the whole, equal parts and fractions was not difficult for the (average or above average) students from area model to the number line
- However, for below average students, it was not straightforward (Figure 4b); determining the equal parts in the number line
- When linking to the sets of objects, the students might be challenged by determining 2/3 in a given sets of objects. Further research needs to confirm this.



(a)



(b)

Figure 4. A student’s work on the task using area model and number line

Findings

Chocolate for outstanding team (Measurement FD)

Teacher has a chocolate bar. It will be given for one outstanding team in class. A serving is 1/5. It remains 9 of 10 slices, 1 slice was already eaten. How many servings can be made?

- It was not a big deal for the students to represent 9/10 and the whole but determining the 1/5 in the PR was demanding; equivalent fractions. This might be also the case for sets of objects
- The quotient requires the students' ability to determine *unit*. It is a challenge in the number line

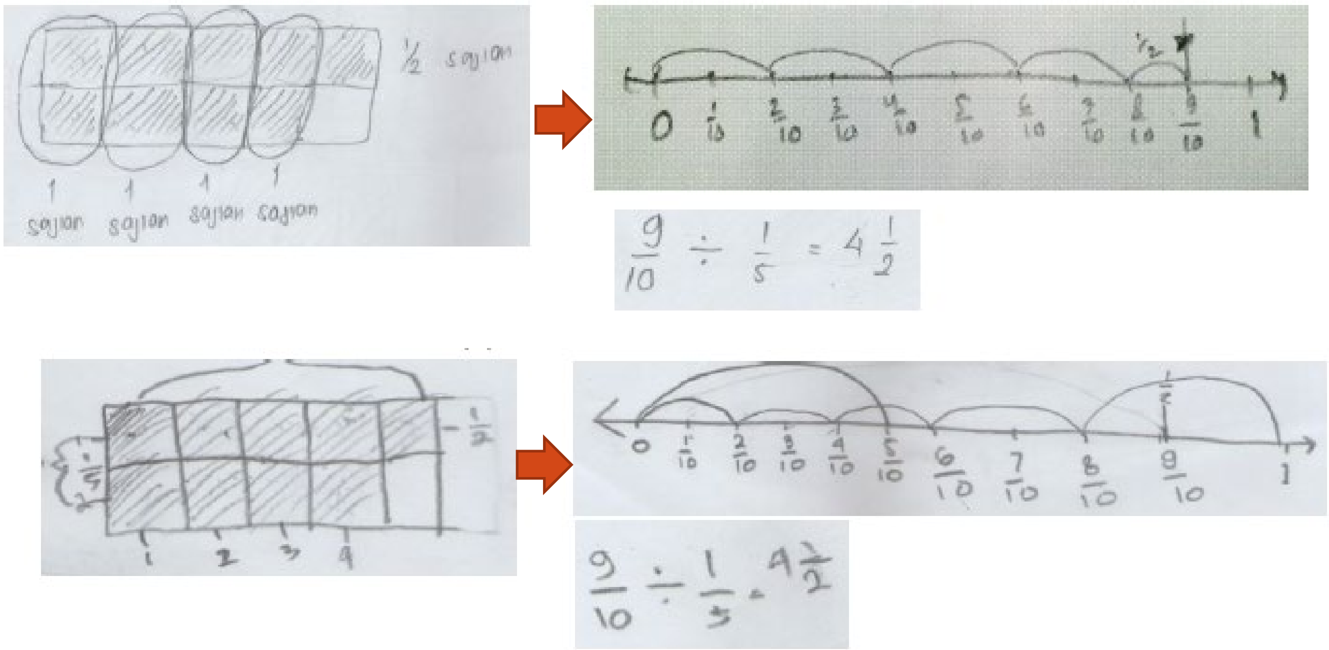


Figure 5. Students linked the area model to number line when solving the task

Some implications

- For type I measurement FD (a ÷ b/c, **a** is divisible by **b,** the quotient is natural numbers) and any context, starting with area model is a good idea (*Wahyu, Amin & Lukito, 2017*). The developed equipartitioning and iterating when using the PR will help students determine the equal parts and linking across the number line and sets of objects.
- In the case of type II measurement FD (a/b ÷ c/d = k, k is natural numbers, b = d), it is not a problem for the students to determine the whole (similar denominators), equal parts and the fraction for each PR. Starting with a PR, which is in line with the context is a good idea.
- Type II measurement FD where b ≠ d poses a challenge for the students in determining the whole due to the different denominators, specifically in number line and sets of objects. Understanding equivalent fractions is required.
- Type III measurement FD (a/b ÷ c/d, the quotient is fraction) involves *unit.* Representing *unit (*comparing the *leftover* with the *whole*) across PR is a challenge for students.

Some implications

- For the case of partitive FD ($a/b \div c$, $a \neq c$) with non-cake context (e.g., hour context), the use of sets of objects as a starting point to define the whole, equal parts and fraction is promising. It will also help the students define the three in the new PR (area model and number line). The ideas of equipartitioning and iterating developed in the initial model are decisive for linking to other PR.
- For partitive FD ($a/b \div c$, $a = c$), the whole, equal parts and fractions can be easily determined in the PR
- Further research is required to understand how students work with multiple PR involving $a/b \div c/d$. Early insight on how primary students solve partitive FD (Wahyu et al., 2020).
- Overall, the contexts and the features of each PR are the factors that support students link across multiple PR in solving FD problems

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