Specialized fractions division knowledge: A case of Anna

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Introduction

- Wahyu (2020) proposed a model of Specialized Fractions Division Knowledge (SFDK, Figure 1) which is built on Specialized Content Knowledge (Ball, Thames, & Phelps, 2008), the conceptualizations of fraction division (e.g., Sinicrope, Mick, & Kolb, 2002), and the representations of fractions division (e.g., Adu-Gyamfi et al., 2019).

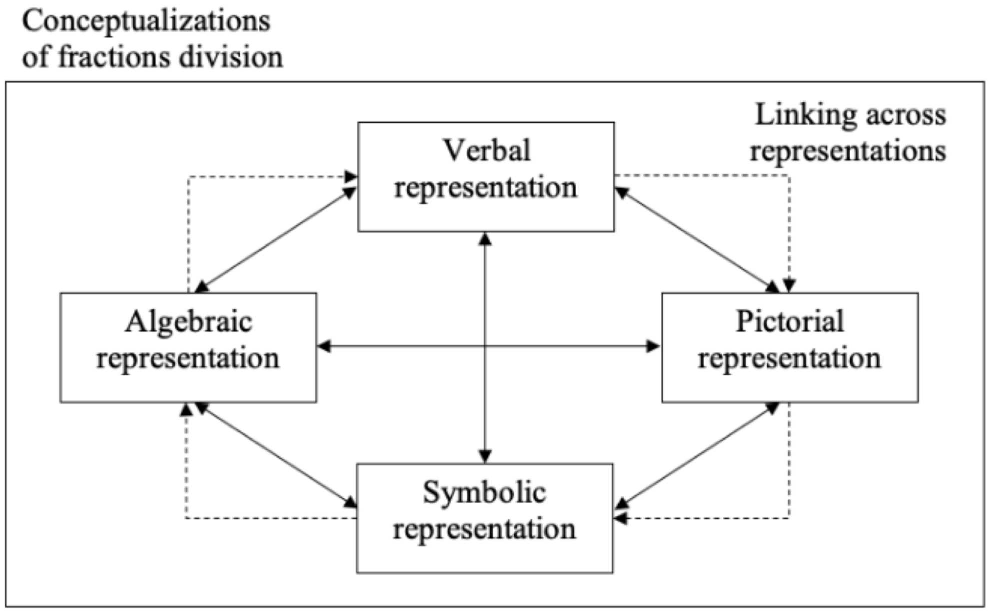

Figure 1. A model of specialized fractions division knowledge (Wahyu, 2020)

Introduction

- Figure 1 demonstrates a connected and flexible SFDK. The first is PTs' ability to translate across various representations, not only from verbal to pictorial representations or one direction translation. The latter is their capability of differentiating conceptualizations of fractions division which affect their works on the representations (Wahyu, 2020).
- It is argued that the model has dual function; to examine and develop PTs' SFDK in the teacher education program.
- This video pitch presents the works of a prospective primary teacher (Anna) which reveal how the model works to examine the natures of PTs' SFDK.

Method

Participants and data collection

- Sixty-four PTs (7 males and 57 females) enrolled in a content mathematics course were given a set of mathematics task which aims to reveal their nature of SFDK. Six PTs were purposively chosen to be interviewed.
- In this video pitch, the works and results of interview of one participant is presented.

Method

Mathematics tasks

- The task comprises five problems which pertain to the model.

Task	Description	Aims
1	Matching number sentences with word problems of fractions divisions	Examining how PTs determine number sentences (symbolic representation) from a given word problem of fractions division (verbal representation). *verbal ↔ symbolic*
2	Solving word problems of fractions division using models and algorithm	Examining PTs' self-generated solution using models and algorithm-based answers. *verbal ↔ pictorial* and *verbal ↔ symbolic ↔ algebraic ↔ verbal*
3	Identifying and reasoning on the linkage of models and algorithm-based answers	Examining PTs' knowledge of linking across representations. *verbal ↔ pictorial ↔ symbolic ↔ algebraic ↔ verbal*
4	Determining problem-context for the given number sentences of fractions division	Examining PTs' knowledge of making different problem context. *symbolic ↔ verbal*
5	Reasoning on the difference of problem contexts relating to number sentences	Examining PTs' knowledge of differentiating conceptualizations of fractions division.

Method

Sample task

- Hint: Use contextual problems in number 1. You can compare it to your contextual problems in number 4. What is the difference of contextual problems for $4 \div \frac{2}{3}$ and $\frac{2}{3} \div 4$? (Task 5)

Data analysis

- Thematic qualitative content analysis was used to analyze the students' works and interview (Kuckartz, 2014; Mayring, 2014)

Findings

Task 1

- (+) Anna is the only one who correctly match the given contextual problems with the number sentences. Sixty-three PTs had reversed order of dividend or divisors and fraction multiplication.
- (-) Anna was difficult to reasoning about the number sentences of the unit rate problems (Interview)

Task 2-3

- Anna was able to construct pictorial representations (area model and number line) in solving four word-problems. ***verbal ↔ pictorial ↔ symbolic ↔ verbal***
- She had errors in determining unit, remainder, and quotient for the remaining three problems.

Figure 2. Anna's errors in determining unit and remainder for word problem 4 and 5

Findings

Task 2-3

- She properly used invert-multiply algorithm
- Her errors resulted in the unmatched answer using models and algorithm. Anna prefer self-generated solutions using models to algorithm ones.
- How self-generated answers using pictorial representations relate to invert-multiply algorithm? or Can the algorithm be justified through the constructed models? Too difficult to be answered by the PTs in the interview. ***pictorial ↔ algebraic***

In the interview…

Anna tried using the way of constructing models for measurement fraction division (problem 5) to solve unit rate fractions division (problem 7). But, she could not accomplish it; Thus, types of fractions division 'guide' the representations (e.g., the constructed models) as depicted in the model.

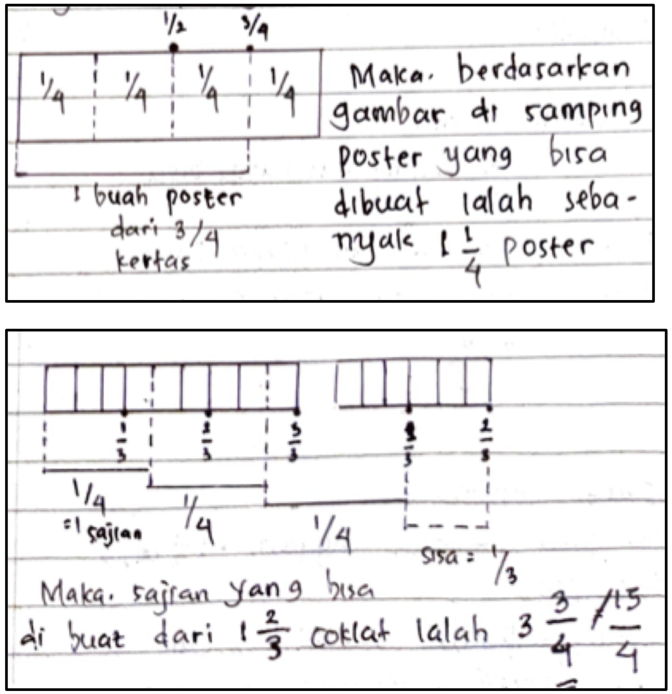

Figure 2. Anna's errors in determining unit and remainder for word problem 4 and 5

Findings

Task 4

- Anna successfully made different contextual problems for $4 \div \frac{2}{3}$ (Measurement); $\frac{2}{3} \div 4$ (Partitive); and $\frac{3}{4} \div \frac{1}{2}$ (Unit rate).
- However, she wrote unrealistic context of measurement fraction division for $1\frac{2}{3} \div \frac{1}{4}$.

In the interview…

Anna realized that the context is unrealistic since it will result in 6 2/3 children. Then she made a new one but still unrealistic.

I had her referred to the given problems.

She came up with 1 2/3 apple will be made juice. One serving (glass) of juice requires ¼ apple. How many servings (glass) of juice can be made?

Findings

Task 5

- Anna identified the difference of contextual problems for $4 \div \frac{2}{3}$ and $\frac{2}{3} \div 4$ from the *words pattern* and *the questions*.
- However, she could not reach episodic situation comprehension (Staub & Reusser, 1995) which leads to the difference of each types of fractions division.

In the interview…

Anna was not sure about her identification about the similarity and difference of fractions division problems.

We discussed episodic situation of each problem. It made her aware of different conceptualization of fractions division (measurement, partitive, unit rate)

What we have from Anna's works on the designed mathematics tasks and interview to help develop PTs' SFDK?

- Develop PTs' ability to determine a number sentence from a problem context which relates to different conceptualizations of fractions division (*verbal ↔ symbolic, mathematics problem model,* Staub & Reusser, 1995) – Task 1 and task 4
- Construct various pictorial representations and link them with symbolic representations (meaningful fractions division, *verbal ↔ pictorial ↔ symbolic ↔ verbal*) – task 2 and task 3
- Develop a meaningful common-denominator and invert-multiply algorithm (Gregg & Gregg, 2007) (*verbal ↔ pictorial ↔ symbolic ↔ algebraic ↔ verbal*) – task 2 and task 3
- Task 1-4 supports connected SFDK. Pre-requisite: Unit, remainder, partition, iteration
- Support PTs' knowledge of differentiating conceptualizations of fractions division by focusing on identifying episodic situation of a word problem. Task 5 supports flexible SFDK

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